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## *Electrical Engineering Fundamentals: AC Circuit Analysis*

Course No: E10-001  
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## ***Preface***

Many Non-engineering professionals as well as engineers who are not electrical engineers tend to have a phobia related to electrical engineering. One reason for this apprehensiveness about electrical engineering is due to the fact that electrical engineering is premised concepts, methods and mathematical techniques that are somewhat more abstract than those employed in other disciplines, such as civil, mechanical, environmental and industrial engineering. Yet, because of the prevalence and ubiquitous nature of the electrical equipment, appliances, and the role electricity plays in our daily lives, the non-electrical professionals find themselves interfacing with systems and dealing with matters that broach into the electrical realm. Therein rests the purpose and objective of this text.

This text is designed to serve as a resource for exploring and understanding basic electrical engineering concepts, principles, analytical strategies and mathematical strategies.

If your objective as a reader is limited to the acquisition of basic knowledge in electrical engineering, then the material in this text should suffice. If, however, the reader wishes to progress their electrical engineering knowledge to intermediate or advanced level, this text could serve as a useful platform.

As the adage goes, “a picture is worth a thousand words;” this text maximizes the utilization of diagram, graphs, pictures and flow charts to facilitate quick and effective comprehension of the concepts of electrical engineering.

In this text, the study of electrical engineering concepts, principles and analysis techniques is made relatively easy for the reader by inclusion of most of the reference data, in form of excerpts from different parts of the text, within the discussion of each case study, exercise and self-assessment problem solutions. This is in an effort to facilitate quick study and comprehension of the material without repetitive search for reference data in other parts of the text.

Due to the level of explanation and detail included for most electrical engineering concepts, principles, computational techniques and analyses methods, this text is a tool for those engineers and non-engineers, who are not current on the subject of electrical engineering.

The solutions for end of the segment self-assessment problems are explained in just as much detail as the case studies and sample problem in the pertaining segments. This approach has been adopted so that this text can serve as an electrical engineering skill building resource for engineers of all disciplines. Since all segments and topics begin with the introduction of important fundamental concepts and principles, this text can serve as a “brush-up,” refresher or review tool for even electrical engineers whose current area of engineering specialty does not afford them the opportunity to keep their electrical engineering knowledge current.

In an effort to clarify some of the electrical engineering concepts effectively for energy engineers whose engineering education focus does not include electrical engineering, analogies are drawn from non-electrical engineering realms, on certain complex topics, to facilitate comprehension of the relatively abstract electrical engineering concepts and principles.

Each segment in this text concludes with a list of questions or problems, for self-assessment, skill building and knowledge affirmation purposes. The reader is encouraged to attempt these problems and questions. The answers and solutions, for the questions and problems, are included under Appendix A.

Most engineers understand the role units play in definition and verification of the engineering concepts, principles, equations, and analytical techniques. Therefore, most electrical engineering concepts, principles and computational procedures covered in this text are punctuated with proper units. In addition, for the reader’s convenience, units for commonly used electrical engineering entities, and some conversion factors are listed under Appendix C.

Most electrical engineering concepts, principles, tables, graphs, and computational procedures covered in this text are premised on SI/Metric Units. However, US/Imperial Units are utilized where appropriate and conventional. When the problems or numerical analysis are based on only one of the two unit systems, the given data and the final results can – in most cases

- be transformed into the desired unit system through the use of unit conversion factors in Appendix B.

Some of the Greek symbols, used in the realm of electrical engineering, are listed in Appendix C, for reference.

**What readers can gain from this text:**

- Better understanding of electrical engineering terms, concepts, principles, laws, analysis methods, solution strategies and computational techniques.
- Greater confidence in interactions with electrical engineering design engineers, electricians, controls engineers and electrical engineering experts.
- A number of skills necessary for succeeding in electrical engineering portion of various certification and licensure exams, i.e. CEM, Certified Energy Manager, FE, Fundamentals of Engineering (also known as EIT, or Engineer in Training), PE, Professional Engineering and many other trade certification tests.
- A better understanding of the many electrical engineering components of typical energy projects.

An epistemic advice to the reader: if you don't understand some of the abstract concepts the first time, don't give up. Read it again! Such is the nature, intrigue and challenge of engineering, physics, science and other subjects that require thinking, reflection and rumination.

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## **Segment 1**

### **Alternating Current (AC) Fundamentals**

#### **Introduction**

This segment begins with an introduction to AC, i.e. alternating current, and segues into a comparison between AC and DC. This comparison accentuates the complexity of AC as compared with DC due to the use of complex numbers and vectors for complete representation of AC entities and parameters. In this segment, we get an opportunity to appreciate how the complexity of AC is amplified by three phase AC consideration in more common, industrial and commercial applications. In an effort to allow readers ample opportunity to explore and learn basic, single and three phase AC principles and mathematical computations, we introduce various single and three phase AC functions, equations and mathematical techniques; reinforced by examples and self-assessments problems. The depth to which the readers may endeavor to explore the topics in this segment depends on their need, appetite and aptitude.

Due to the practical applications of the process of electrodeposition, in the coatings industry - and the innate relationship between electrodeposition and electricity – this process will be defined and illustrated through equations and practical numerical examples. This segment provides the reader an introduction to AC electrical transformers of various types; single phase and three phase. Of course, similar to other topics in this text, the concepts, principles, equations and applications in this segment are illustrated with analogies, numerical examples and end of the segment problems. All in all, this segment provides an adequate introduction to alternating current and prepares the reader for intermediate level study of AC concepts.

#### **Alternating current (AC) versus direct current (DC)**

The contrast between AC and DC isn't just rooted in physics but goes as far back as the 1800's when, for a period of time, both vied for the residential, commercial and industrial markets. Electricity was first discovered and harnessed, mostly, in the DC, or direct current form. However, due to the physical constraints associated with transmission, distribution and application of DC in industrial, commercial and residential applications, AC vanquished DC in the power distribution arena. Nevertheless, today DC holds its own in

low voltage control and digital logic applications. In fact, albeit rare, DC is being reintroduced in the ultra-high voltage realm. The 900,000 Volt DC transmission line system project, currently underway in India, would serve as a more contemporary testimonial in that regard. This ultra-high DC voltage system is expected to redistribute surplus power from India's hydroelectric rich north east region to the energy starved, high growth, central region.

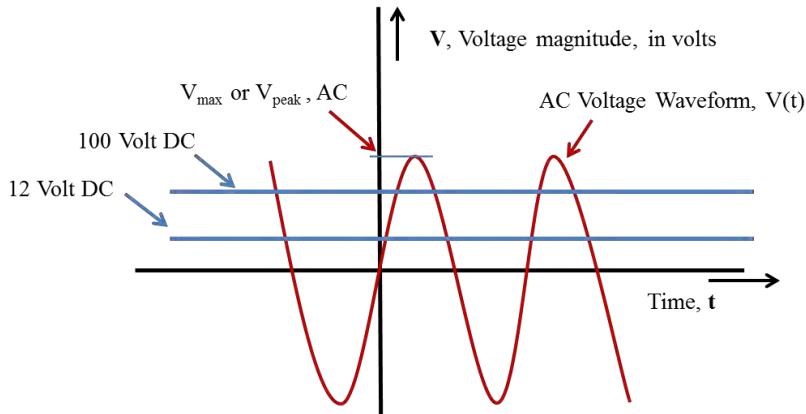
If we could attribute the acceptance of AC for residential, commercial and industrial applications to one pivotal entity, it would be the **transformer** and concept of **voltage transformation**. In essence, when DC was first distributed and transmitted to end users, it was at a fixed voltage level because DC voltage transformation was not feasible. Due to resistance in the transmission lines - as stipulated by Ohm's Law:  $V = IR$  - the DC power generators had to be located in the vicinity of the consumers to mitigate the undesirable voltage regulation issues. Longer transmission runs resulted in higher resistance, or "**R**," which, in turn, resulted in larger voltage drop, "**V**." In the mechanical fluids realm, this would be analogous to unacceptable pressure drop – due to frictional head losses – if fluid is distributed over extended distances without some sort of booster pumps.

Introduction of AC accompanied practical means for "stepping up" and "stepping down" of voltage through the use of transformers. So, AC could be generated at, for instance, 4,000 volts, stepped up to 100,000 volts – through the application of transformers – for transmission purposes, and miles away, it could be stepped down to usable levels, such as, 480 volt, 240 volts or 120 volts. That is, mostly, the way AC is generated, transmitted and distributed to consumers today.

We learnt earlier that movement of electrons, or other charged particles, constitutes electrical current. When the charged particles move in one specific direction – such that there is a net displacement in their position, or when the charged particles travel a net distance – **direct current, or DC** is said to exist. On the other hand, when the electrons or charged particles oscillate or vibrate about a point – similar to a pendulum - such motion of charged particles constitutes **alternating current, or AC**.

If voltage and current were plotted on Cartesian Coordinates, the resulting graphs would be as depicted in Figures 1.1 and 1.2, respectively. In fact, if AC

voltage, DC voltage and current were compared using an *oscilloscope*<sup>1</sup>, the graph displayed on the screen would be similar to voltage and current graphs shown in Figures 1.1 and 1.2, respectively.



**Figure 1.1:** Voltage comparison, AC versus DC

As we examine Figure 1.1, we see that the two horizontal flat lines represent DC voltage, plotted on two dimensional X-Y coordinates. The lower flat line represents a typical 12 volt automotive battery. The upper flat line, labeled 100 Volt DC, represents the DC equivalent of 110  $V_{RMS}$ , or 156  $V_{peak}$  – i.e. the typical US electrical outlet voltage. The 100 Volt DC equivalent of 110  $V_{RMS}$  applies to the DC power derived through full wave rectification of AC input power. The conversion of AC to DC and associated computations are illustrated through Example 1.1 and equations introduced in the electrodeposition section.

Further exploration of Figure 1.1 reveals that the x-axis (or abscissa) represents time,  $t$ , in seconds. The Y-axis (or ordinate) represents the voltage magnitude,  $V$ , in volts. The sine waveform, oscillating about the time axis, represents the AC voltage, varying as a function of time,  $t$ .

DC and AC *currents* can also be contrasted using graphical representations, in a manner similar to the DC and AC voltage comparison conducted above.

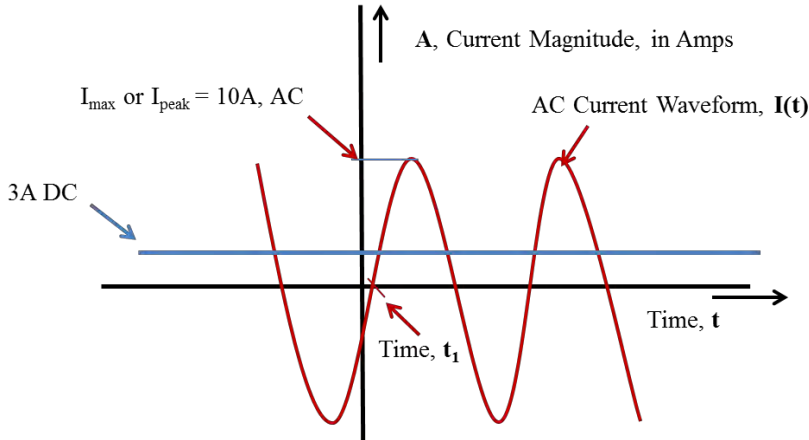
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<sup>1</sup>*Oscilloscope* is an electrical instrument used to graphically view and analyze voltage, current, frequency or power performance of electrical or electronic systems or circuits.



The DC and AC **current** functions are depicted in Figure 1.2 in a form that is similar to their voltage counterparts. The horizontal flat line represents the DC current set up by virtue of the DC voltage of the DC power source; which can be a battery or a DC power supply. The Y-axis of this **current vs. time** plot represents the current magnitude, **I**, in Amps. The DC current, in this case, is assumed to be a constant 3A.

The sine waveform, oscillating about the time axis, represents the AC current,  $I(t)$ , varying as a function of time,  $t$ . The peak or maximum value of the AC current, in this case is assumed to be 10A. Note that in this illustration, the AC current appears to be surfacing into the positive current territory at time  $t_1$ . If one were to assume that this current was produced or driven by the voltage depicted in Figure 1.1, where the voltage broaches through the x-axis at time  $t = 0$ , one could say that the current is *lagging behind the voltage*. As we will explore later in greater depth, in the power factor section, such a situation where current lags behind the voltage is said to cause a **lagging power factor**.



**Figure 1.2:** Current comparison, AC versus DC

### Electrodeposition - DC and the Average Value of AC

Electrodeposition, as mentioned earlier, plays an important role in the coatings operations. Simply put, it involves the use of electrical charge for the deposition of paint pigments to metallic surfaces. When electrical charge is used to transport paint pigments to a metallic surface, the pigments tend to penetrate and adhere to the metal surface more strongly, uniformly and durably than if the surface were painted through the traditional spray or brush painting methods. Since electro deposition process involves net displacement of relocation of the pigments, DC electrical current and voltage have to be employed. AC voltage, due to the periodic oscillation or reversal of voltage in the positive and negative realms, would **not** deliver the net displacement of the pigment particles.

In electro deposition or electroplating scenarios, often the amount of electroplating required is specified in form of the amount of charge that is needed to be transferred in order to achieve a certain thickness of electroplating or electrodeposition. The basic equation that is useful in determining how **long** a known amount of DC current must be passed in order to achieve the required charge transfer is:

$$I_{DC} = \frac{q}{t} , \text{ or, } t = \frac{q}{I_{DC}} \quad \text{Eq. 1.1}$$

Where, **q** = charge in coulombs of faradays, **I<sub>DC</sub>** = DC current and **t** = time in seconds. Note that symbols “q” and “Q” may be used interchangeably to denote charge.

Conversion of AC voltage into DC voltage, accomplished through the full wave rectification, can be quantified using the following equation:

$$V_{DC} = 2 \cdot \left( \frac{V_{max}}{\pi} \right) \quad \text{Eq. 1.2}$$

Where,

$$V_{max} = \sqrt{2} \cdot V_{rms} \quad \text{Eq. 1.3}$$

In order to derive the DC current needed to compute the duration of electrodeposition, the effective resistance of the plating tank, with paint – or liquid pigment solution – is needed. This effective resistance can be measured. With the effective resistance known, the DC current can be calculated using the Ohm's law:

$$I_{DC} = \frac{V_{DC}}{R} \quad \text{Eq. 1.4}$$

A common computation associated with application of AC and DC in the electrodeposition process is illustrated in Example 1.1.

### **Example 1.1**

A plating tank with an effective resistance of 150 ohm is connected to the output of a full-wave rectifier. The AC supply voltage is  $120V_{rms}$ . Determine the amount of time it would take to perform 0.01 faradays worth of electroplating?

#### **Solution:**

**Background/Theory:** The amount of coating or electroplating being accomplished in this example is specified in terms of the electrical charge that must be moved or transferred; which in this case is specified in Faradays.

**Faraday** is a unit for quantifying electrical charge, similar to **Coulomb**.

$$1 \text{ amp} = 1 \text{ Coulomb/sec}$$

Or,

$$1(A).(s) = 1 \text{ C}$$

And,

$$96,487 \text{ Coulomb} = 1 \text{ faraday}$$

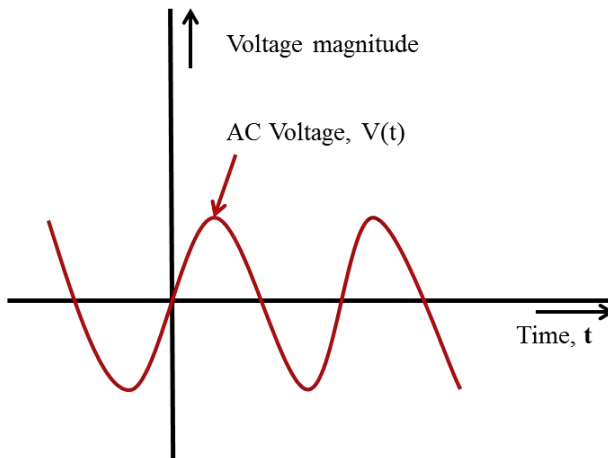
Therefore, the amount of charge transfer in Coulombs, in this electrodeposition case, would be:

$$q = \left( \frac{96,487 \text{ Coulombs}}{1 \text{ Faraday}} \right) \cdot (0.01 \text{ Faraday}) = 964.87 \text{ Coulombs}$$

Since we are interested in the amount of time it takes to transfer a known amount of charge, rearrangement of Eq. 1.1 results in:

$$t = \frac{q}{I}$$

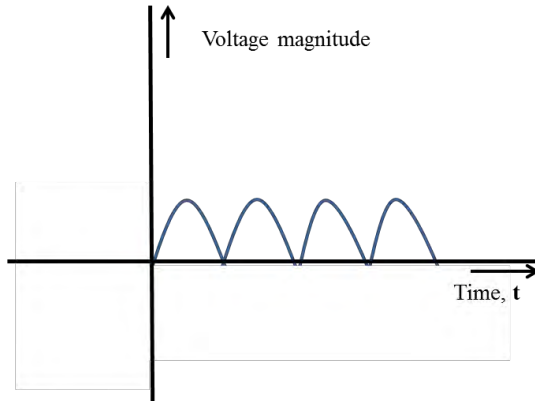
The next step entails determination of the DC current produced by the full wave rectification of 120 V<sub>max</sub> AC. The net effect of full wave rectification of AC into DC is illustrated through the graphs depicted in Figures (a) and (b), below. The graph in Figure (a) shows the AC waveform on the **input** side of a full wave rectifier.



**Figure (a)**

The graph in Figure (b) shows the **DC output** side of a full wave rectifier; the net effect being the “flipping” or reversing of the negative troughs of the AC waveform such that the resulting output waveform has a definite **average value**. Note that the average of the sinusoidal AC waveform is **zero** because the AC voltage or current dwells the same amount of time in the positive realm – or above the x-axis - as it does in the negative realm, or below the x-

axis. In other words, on the AC side, since the total **positive** area portended by the voltage or current graphs equals the corresponding **negative** areas on the **AC input** side of the rectifier, the average value is zero, and the DC content - voltage or current - on the AC input side is **zero**.



**Figure (b)**

Based on the data provided, we must first convert the given  $V_{\text{rms}}$  voltage into  $V_{\text{max}}$  using Eq. 1.3, and then compute  $V_{\text{DC}}$  by applying Eq. 1.2. As shown below, these two steps are followed by the application of Ohms Law, in the form of Eq. 1.4, to compute the DC current,  $I_{\text{DC}}$ .

$$V_{\text{max}} = \sqrt{2} \cdot V_{\text{rms}} = (1.414) \cdot (120\text{V}) = 170 \text{ V} \quad \text{Eq. 1.3}$$

$$V_{\text{DC}} = 2 \cdot \left( \frac{V_{\text{max}}}{\pi} \right) = 2 \cdot \left( \frac{170}{3.14} \right) = 108 \text{ V} \quad \text{Eq. 1.2}$$

And,

$$I_{\text{DC}} = \frac{V_{\text{DC}}}{R} = \frac{V_{\text{Ave}}}{R} = \frac{108}{150} = 0.72\text{A} \quad \text{Eq. 1.4}$$

Then, application of Eq. 1.1 yields:

$$t = \frac{964.87 \text{ Coulombs}}{0.72 \text{ Coulombs/sec}} = 1340 \text{ seconds}$$

## **Alternating Current and Impedance**

Impedance is the current opposing or current impeding characteristic of a load (or conductor) in an AC circuit. As implied in the definition of this term, impedance is an alternating current (AC) entity. While in DC circuits the factor that opposes the flow of current is resistance, the entity that influences the flow of AC current is impedance. Impedance, like AC current, voltage and power, is a vector entity and, therefore, it can be fully defined by two key characteristics, the *magnitude* and the *direction*.

## **Mathematical and Geometric Representation of Complex AC Entities**

Vector, complex number based entities, such as impedance, AC current and voltage can be expressed or quantified in the following forms:

- a) Polar or Phasor form
- b) Rectangular form
- c) Sinusoidal form
- d) Exponential form

### **Polar or Phasor Form**

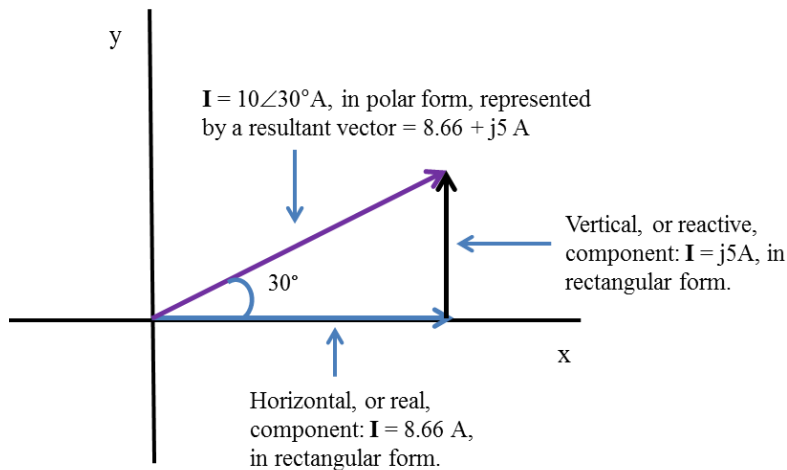
The terms polar and phasor are used synonymously. Phasor or polar representation of AC entities such as impedance, current, voltage and power requires definition of the magnitude of the respective entity and the direction in degrees (°). The polar/phasor representation of *AC current* would be represented as follows:

$$\mathbf{I} = I_m \angle \theta^\circ \quad \text{Eq. 1.5}$$

Where,  $\mathbf{I}$  is the vector or complex value of AC current. The magnitude of the overall AC current, unless otherwise specified, is the maximum or peak value, and is denoted by  $I_m$ . In the phasor representation of AC current above,  $\theta^\circ$  is the angle of the AC current.

As an example of phasor/polar representation of AC current, consider a current  $\mathbf{I} = 10 \angle 30^\circ$  A. In this phasor representation of AC current, 10 represents the peak or maximum *magnitude* of AC current in amps, and  $30^\circ$  represents the *angle* of the current. This is illustrated in Figure 1.3. This

perception or understanding of AC current, as a vector, is analogous to the role of a **force** vector in mechanical or civil engineering realms; such as, in the study of statics, where, the magnitude of a vector entity such as force may be defined in Newtons (N) or pounds force (lbf) and the direction of the force in degrees.



**Figure 1.3:** Vector representation of phasor value of AC current

Note that in most practical applications and computations, the root mean square value of current, or  $I_{\text{rms}}$ , is used instead of the maximum/peak magnitude of current  $I_{\text{max}}$ . For instance, the overcurrent protection devices, i.e., fuses, breakers – and circuit isolation apparatus like the disconnect switches – to name a few, are specified in rms terms. The same is true for AC voltages.

## Rectangular Form

Rectangular representation of AC entities such as impedance, current, voltage and power entails numerical definition of those entities in form of horizontal and vertical, vector, components. An AC current of  $10\angle 30^\circ$  A rms, represented in the polar or phasor form, can be translated to the corresponding rectangular as:  $8.66 + j5$  A rms; where the horizontal component of 8.66A represents the “real” component of the overall AC current and 5A represents the “imaginary” or “reactive” component of the overall AC current. This conversion of the AC current from its phasor to rectangular form can be accomplished through a scientific calculator or by performing trigonometric

calculations. The trigonometric approach would involve the computation of horizontal and vertical components of the rectangular form, as follows:

$$I_{\text{real}} = 10\cos 30^\circ = 10(0.866) = 8.66 \text{ A}$$

And,

$$I_{\text{reactive}} = 10\sin 30^\circ = 10(0.5) = 5 \text{ A}$$

In the mechanical engineering or kinetics realm, the rectangular representation of AC current would be similar to the representation of a  $10\angle 30^\circ$  mph velocity vector in the form of its horizontal and vertical components as 8.66 mph and 5 mph, respectively.

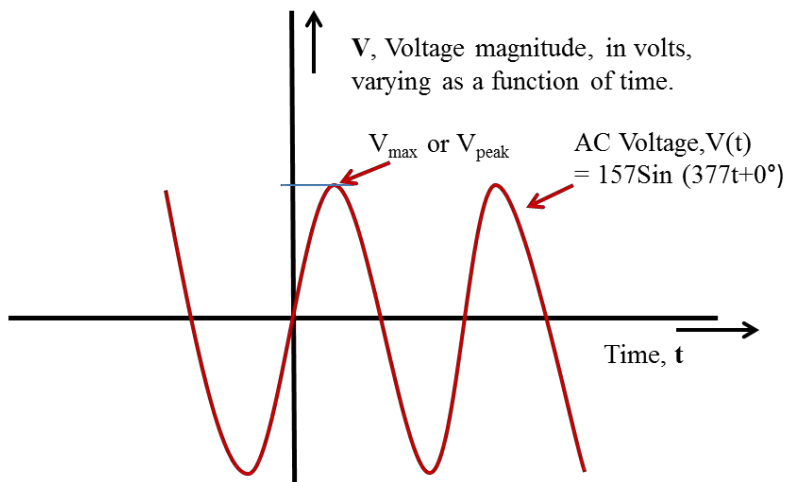
### **Sinusoidal or Trigonometric Form**

The sinusoidal representation of AC parameters or entities involves the application of trigonometry. Sinusoidal representation of an AC entity, such as AC voltage, would be as follows:

$$V = V(t) = V_m \sin(\omega t + \theta)$$

**Eq. 1.6**

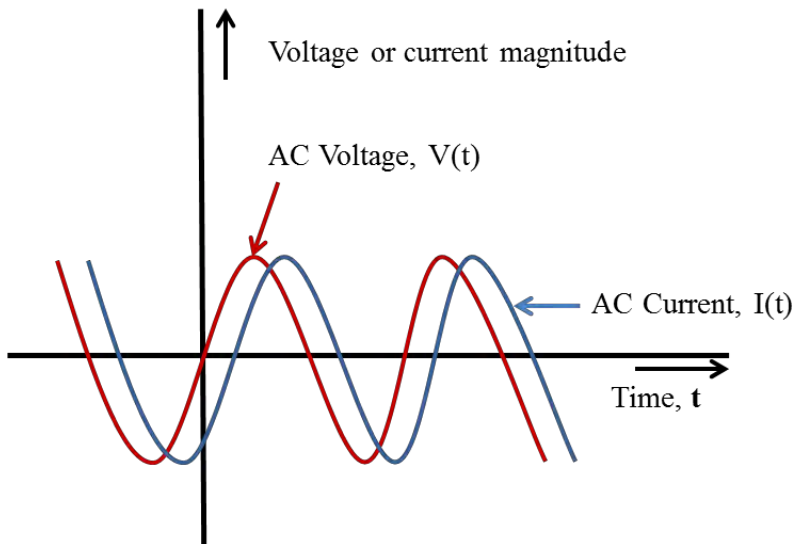
Where, “**V**” and  $V(t)$  denote sinusoidal AC voltage,  $V_m$  is the maximum or peak voltage,  $\omega$  represents the angular frequency, in rad/sec, and  $\theta$  represents the angle of the AC voltage, in degrees. Figure 1.4, below, depicts this AC voltage function in the graphical form.



**Figure 1.4** Graphical representation of sinusoidal voltage



Note that the AC current  $I$ , or  $I(t)$ , would be represented by a graph, very similar to the voltage  $V(t)$  graph, with the exception of the fact that the  $I(t)$  wave form would be shifted to the left or right of the voltage wave form, depending upon whether the reactance of the AC load is predominantly inductive or capacitive. This is illustrated in Figure 1.5 by a sinusoidal voltage and current graph of a scenario with predominantly inductive load; where, the current is lagging the voltage.



**Figure 1.5** Graphical contrast between AC voltage and current wave forms

Continuing with our example of the  $10\angle 30^\circ$  A rms AC current, we can illustrate its sinusoidal AC form by converting the phasor AC rms current value into its sinusoidal form as:

$$I(t) = I_{\text{rms}} \text{Sin} (\omega t + \theta^\circ) \text{ A-rms, in the general form,} \quad \text{Eq. 1.7}$$

And,

$$I(t) = 10 \text{ Sin} (377t + 30^\circ) \text{ A-rms, in this specific case.}$$

Since,

$$I_{\text{max}} = (\sqrt{2}) \cdot I_{\text{rms}} = (1.41) \cdot (10) = 14.1\text{A}$$

In peak or maximum value form, the sinusoidal representation would be as follows:

$$I(t) = 14.1 \sin(377t + 30^\circ) \text{ A}$$

Where, the coefficient 14.1 represents the *maximum value* of the current, in amps (A), and  $30^\circ$  represents the angle of the current. The 377 coefficient of time “t” represents,  $\omega$ , the angular frequency of AC in radians per second. This value of 377 rad/sec is derived from the standard US alternating current source frequency of 60 Hz, or 60 cycles/sec. The AC frequency in Great Britain and some of the former British colonies is 50 Hz. The conversion from 60 Hz to 377 rad/sec can be explained mathematically as follows:

$$\begin{aligned} \omega &= 2\pi \cdot f && \text{Eq. 1.8} \\ &= 2 \cdot (3.14) \cdot (60) = 377 \text{ rad/sec} \end{aligned}$$

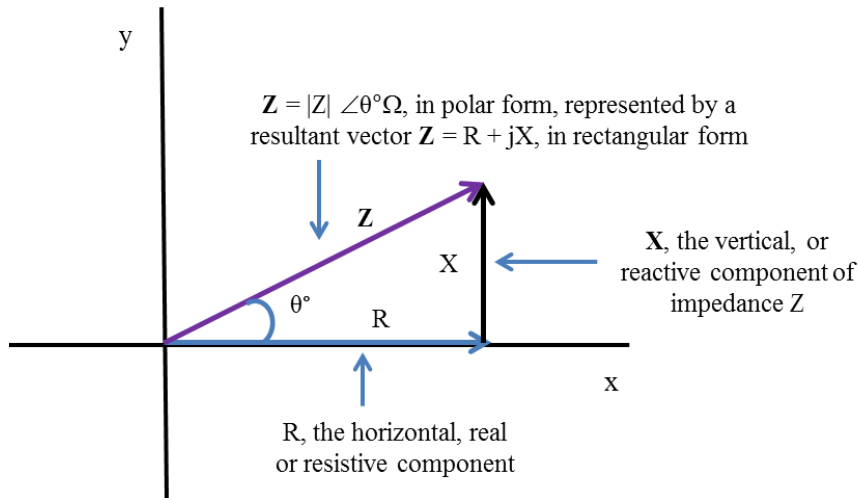
### **Exponential Form**

The exponential representation of an AC parameter – whether it is current, voltage, impedance or power – is somewhat similar in form to the polar/phasor form. This is because the exponential form consists of the maximum value and the angle. For instance, the  $14.1 \angle 30^\circ$  amp AC current example we discussed earlier would be represented in the exponential form as follows:

$$\begin{aligned} \mathbf{I} &= I_m e^{j\theta} && \text{Eq. 1.9} \\ &= 14.1 e^{j30^\circ} \text{ A} \end{aligned}$$

### **Impedance Analysis**

In this section, we will cover mathematical and graphical representation and basic computational methods pertaining to impedance. Because impedance is a vector, it can be drawn or depicted as an arrow whose length represents the magnitude of the impedance while its orientation, expressed in degrees, represents the angle of the impedance. Since a vector can be analyzed or split into its horizontal and vertical components, impedance “Z” can be drawn in the vector diagram format as shown in Figure 1.6.



**Figure 1.6:** Graphical representation of impedance  $Z$  in vector form

When an AC circuit consists of resistance, capacitance and inductance, impedance  $Z$  can be stated mathematically in form of Equations 1.10 and 1.11.

The unit for impedance is **ohm**, or  $\Omega$ ; similar to the unit for resistance,  $R$ , capacitive reactance,  $X_c$ , and inductive reactance,  $X_L$ . Consistency of units between  $R$ ,  $X_L$ ,  $X_C$  and  $Z$  is a requisite for the following mathematical definitions for  $Z$  to hold true:

$$Z = R + Z_L + Z_c \quad \text{Eq. 1.10}$$

$$Z = R + jX_L - jX_c \quad \text{Eq. 1.11}$$

It's obvious from examination of the above two equations that:

$$Z_L = jX_L \quad \text{Eq. 1.12}$$

$$Z_c = -jX_c \quad \text{Eq. 1.13}$$

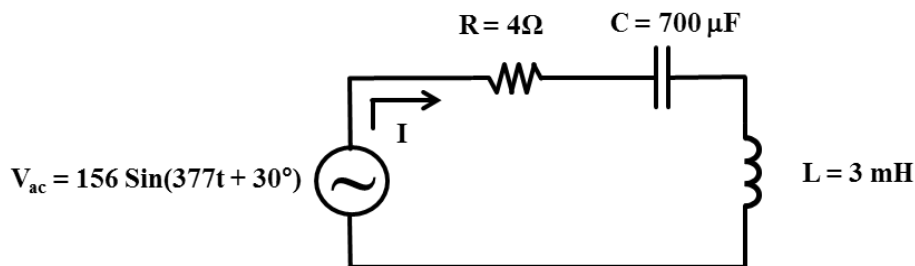
Where,  $Z_L$  is the impedance contribution by the inductance in the AC circuit and  $Z_c$  is the impedance contribution by the capacitance in the AC circuit. The units for all impedances are **ohms**, or  $\Omega$ .

Complete comprehension of AC circuit analysis and associated computations requires basic appreciation of how AC circuit components, such

as resistances, capacitances and inductances, are converted into their respective impedance contributions before they are consolidated into an overall equivalent or combined impedance. In most cases, the derivation of other AC circuit parameters, such as current, voltage and power, is undertaken after determination of the total or equivalent impedance.

**Example 1.2**

Determine the equivalent, or total, impedance  $Z_{Eq}$  and the source rms current,  $I$ , in the AC series circuit below.



**Solution:**

According to Eq. 1.10 and Eq. 1.11:

$$Z = R + Z_l + Z_c \quad \text{Eq. 1.10}$$

$$\text{Or, } Z = R + jX_l - jX_c \quad \text{Eq. 1.11}$$

$$\begin{aligned} \therefore Z_{Eq} &= R + \frac{1}{j\omega C} + j\omega.L \\ &= R + \frac{1}{j(2).(\pi).(f).(C)} + j(2).(\pi).(f).(L) \\ &= 4 + \frac{1}{j(2).(3.14).(60).(700 \times 10^{-6})} + j.(2).(3.14).(60).(3 \times 10^{-3}) \\ &= 4 + \frac{1}{j(0.2638)} + j.(1.1304) \\ &= 4 - j(3.79) + j1.1304 = 4 - j2.66 = 4.8 \angle -33.63 \Omega \end{aligned}$$

Current “**I**” calculation:

$$V(t) = V_{\max} \sin(\omega t + \theta) = 156 \sin(377t + 30^\circ) \quad - \text{ Given}$$

$$\therefore |V_P| = |V_{\max}| = 156 \text{ V}$$

$$\text{And, } |V_{\text{rms}}| = \frac{156}{\sqrt{2}} = 110 \text{ V}$$

$$\therefore V(t), \text{ in rms form, would be } = V_{\text{rms}} \sin(\omega t + \theta) = 110 \sin(377t + 30^\circ)$$

$$\text{And, } V_{\text{rms}}, \text{ in polar or phasor form would be } = 110 \angle 30^\circ V_{\text{rms}}$$

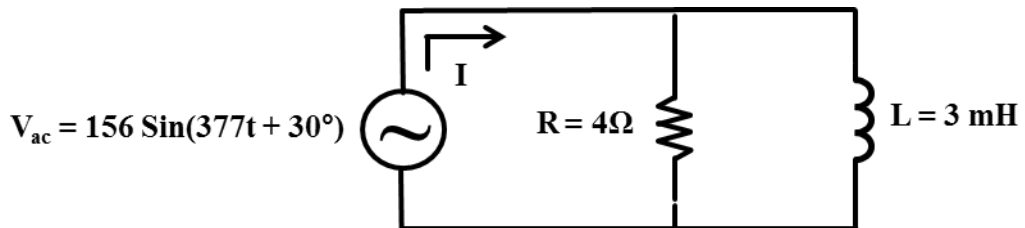
$$\text{Then, according to Ohm's Law, } I = \frac{V}{Z_{\text{Eq}}} = \frac{110 \angle 30^\circ}{4.8 \angle -33.63}$$

$$\text{Or, } I = 23 \angle \{30^\circ - (-33.63)\}$$

$$\therefore \text{ The source rms current, } I = 23 \angle 63.63^\circ \text{ A}$$

### Example 1.3

Determine the equivalent, or total, impedance  $Z_{\text{Eq}}$  and the **rms** value of the source current  $I$  in the AC parallel circuit below.



### Solution:

Solution strategy: Convert the given inductance value of  $L = 3 \text{ mH}$  into its equivalent inductive reactance  $X_L$ , and subsequently into its impedance contribution  $Z_L$ . Then, combine the resistance  $R$  and  $Z_L$  values into the equivalent impedance  $Z_{\text{Eq}}$  by applying the formula for combination of parallel circuit elements.

$$X_L = \omega.L = (377).(3 \times 10^{-3}) = 1.13 \Omega$$

$$\therefore Z_L = j1.13 \Omega = 1.13 \angle 90^\circ \Omega$$

The parallel combination formula is:

$$\frac{1}{Z_{Eq}} = \frac{1}{R} + \frac{1}{Z_L}$$

$$\text{Or, } Z_{Eq} = \frac{(R).(Z_L)}{R + Z_L} = \frac{(4).(1.13 \angle 90^\circ)}{4 + j1.13} = \frac{(4.52 \angle 90^\circ)}{4 + j1.13}$$

$$Z_{Eq} = \frac{4.52 \angle 90^\circ}{4.16 \angle 15.77} = 1.09 \angle 74.22^\circ \Omega$$

Current “I” calculation:

V(t), in rms form, =  $110 \angle 30^\circ V_{rms}$  as determined in the previous example.

$$\text{Then, according to Ohm's Law, } I = \frac{V}{Z_{Eq}} = \frac{110 \angle 30^\circ}{1.09 \angle 74.23}$$

$\therefore$  The source rms current,  $I = 101 \angle -44^\circ A$

## **Transformers**

Earlier in this segment, we introduced the concept of transformers and the historically pivotal role transformers played in the predominant acceptance of AC over DC. In this section, we will explore the technical principles, applications and computations associated with single phase transformers. We will touch on some fundamental characteristics of three phase transformers, as well.

Let's begin by exploring the construction of a simple, single phase, AC transformer based on our first introduction of magnetic flux earlier. We will build upon the toroidal core example we introduced earlier by adding an AC voltage source to the conductor coiled onto the left side of the core, as shown in Figure 1.7, and by winding  $N_s$  number of turns of a conductor on the right side of the same toroidal core. This converts the toroidal core into a basic transformer with  $N_p$  number of turns coiled on the AC source side and  $N_s$  turns on the side that is connected to the load.

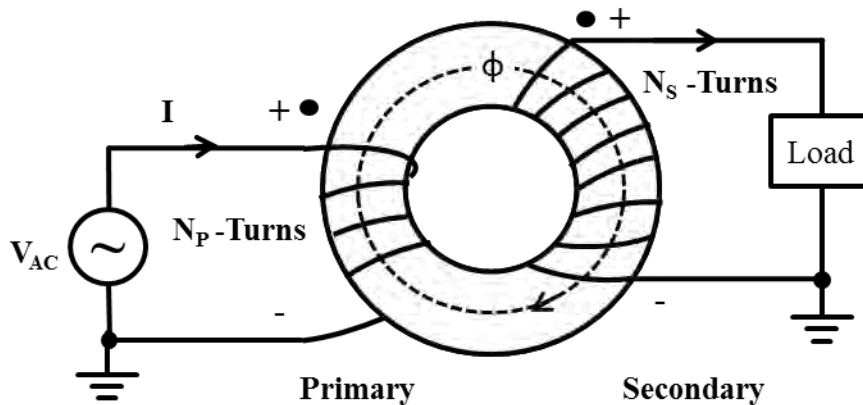


Figure 1.7

**Primary and Secondary Side of a Transformer:** The side or segment of the transformer that is connected to the power source is referred to as the *primary* side. The segment of the transformer that is connected to the load is called the *secondary* side. Note that the toroidal transformer depiction in Figure 1.7 above is simplified for voltage transformation principle illustration purpose. In actual, practical and commercial applications, the primary and secondary windings of a *toroidal core transformer* are concentric, with the core in the center as shown in the Figure 1.8 below.

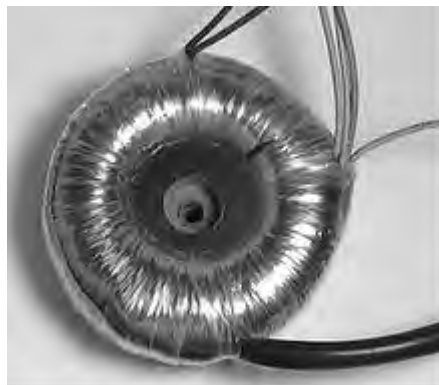
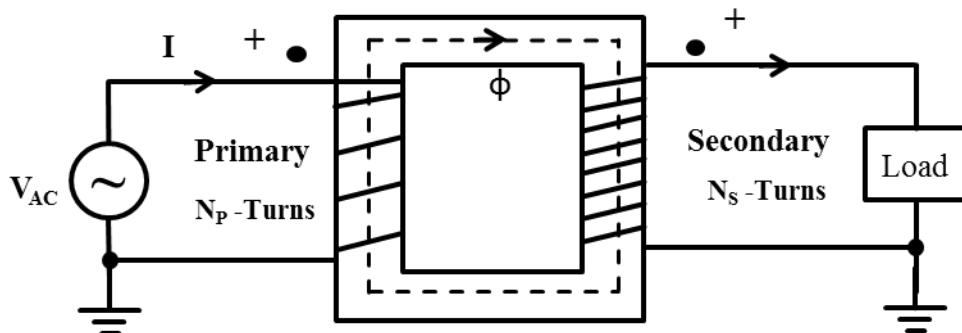


Figure 1.8: Toroidal core transformer by Omegatron ®

Even though we used a toroidal core in this basic transformer example, we could have used a more common rectangular or square core which would

make this transformer resemble an ordinary transformer as shown in Figure 1.9 below.



**Figure 1.9:** Ideal transformer with complete primary and secondary circuits.

As we examine figures 1.7 and 1.9, we notice that one of the voltage source ( $V_{AC}$ ) terminals is grounded. This permits the other terminal to serve as the higher potential, or “hot,” terminal. Hence, the current flows from left to right, or in the clockwise direction through the primary circuit of the transformer. Applying the right hand rule, would result in a clockwise magnetic flux  $\phi$  through the core. On the secondary side, through converse application of the right hand rule, the clockwise core flux  $\phi$  results in the induction of current emerging from the top left side of the secondary circuit. This induced current flows in a clockwise fashion through the load as shown in Figure 1.9. Note that the direction of flow of AC current, on the primary or secondary sides of the transformer, signifies the *direction of flow of AC power or AC energy*, from the source to the load.

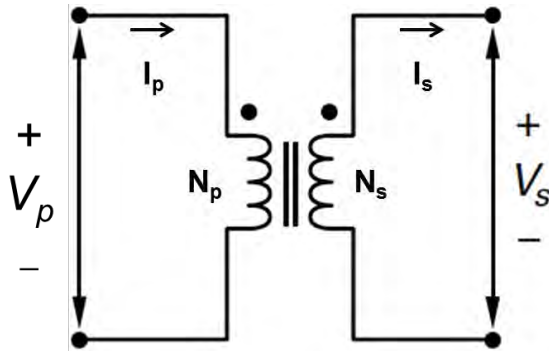
**The Dot Convention for Transformers:** The dots shown, in Figures 1.7 and 1.9, on the primary and secondary sides of the transformer, indicate the *direction of each winding relative to the others*. Voltages measured at the dot end of each winding are in phase. In other words, *they have the same angle*. By convention, the current flowing into the *dot end of a primary coil* will result in current flowing out of the *dot end of a secondary coil*.

**Ideal vs. Real Transformers:** A thorough and comprehensive study of “*real*” transformers is complex. Core and winding losses in most *real* transformers constitute a small percentage – less than 5% – of the total power



vcpuhqtg gf 0Vj gthqtg. 'hqt'uko r nek\{. 'vcpuhqtg gt "ektewk'cpcn\ ugu'ctg'qhvgp" eqpf wevgf "qp'vj g'r tgo kug'vj cv'vj g\{ 'ctg'ōidealō"cpf "ōlosslessō"Vj ku'o gcpu'vj cv'vj g'r qy gt hgf "kpq'cp'kf gcn'vcpuhqtg gt. "qp'vj g'r tko ct\{ "ukf g. "ku'gs wcn'vq'vj g" r qy gt'r w'qww'qp'vj g'ugeqpf ct\{ "ukf g0k'vj ku'vgzv'y g'y knrko kv'vj g'ueqr g'qh'qwt" f kuewukqp'vq'kf gcn'vcpuhqtg gt. "y kj "Iwuv'c'dt'kgh'kpv'qf wevkqp'vq'vj g'eqpegr v' qh'eqtg'cpf "y kpf kpi "hquugu'cuuqekcvf "y kj "tgcn'vcpuhqtg gt u0"

Cp'kf gcn'vcpuhqtg gt "ecp'dg'o qf gngf "cu'uj qy p'kp'Hki wtg'302'dngmy 0'



Hki wtg'302'kf gcn'vcpuhqtg gt 'b qf gnl'

"Vj g'y q'r ctcngn'xgt v'ecn'kpgu. "kp'vj g'o kf frg'qh'vj g'kf gcn'vcpuhqtg gt" f kci tco "uj qy p'kp'Hki wtg'302. "tgr tgu'p'vj g'cuuwo r vkp'vj cv'vj g'eqtg'qh'cp" kf gcn'vcpuhqtg gt "eqpukuu'qh'c'htttqo ci p'gk'qt'bo ci p'gk'bo cvgt'krf"

"Uqo g'qh'vj g'hqto wcu'vj cv'i qxgtp'tgr'vkpuj k' u'dgy ggp'xct'kqu'dcuke" qr gtc'vkpcnr cteo gvtu'qh'cp'kf gcn'vcpuhqtg gt "ctg'cu'hqmy u"

Gs 0304

$$V_s = \frac{V_p}{a} = \left( \frac{N_s}{N_p} \right) \cdot V_p$$

"Gs 0305"

$$I_s = a \cdot I_p = \left( \frac{N_p}{N_s} \right) \cdot I_p$$

"Gs 0306"

$$|S_{1-\Phi}| = V_s \cdot I_s = V_p \cdot I_p$$

Gs 0307"

Where,

$N_p$  = Number of winding turns on the primary side of the transformer

$N_s$  = Number of winding turns on the secondary side

$V_p$  = Voltage applied on the primary side of the transformer

$V_s$  = Voltage on the secondary side of the transformer

$I_p$  = Current flowing through the primary side of the transformer

$I_s$  = Current flowing through the secondary side of the transformer

$S_{1-\phi}$  = Single phase AC power being transmitted by the transformer

Since turns ratio  $a$  represents a *ratio* of the primary turns to secondary turns, it is unit-less. Voltages and currents referred to, typically, in transformer circuit analysis and specifications, are in RMS form. Also, when transformer voltages and currents are measured using voltmeters and clamp-on ammeters, respectively, the resulting measurements are in RMS form. This are the reasons why many brands of voltmeters and ammeters are labeled as *True RMS Voltmeters* and *True RMS Ammeters*, respectively. In Eq. 1.15,  $S_{1-\phi}$  represents the single phase AC power; or, to be more accurate, AC apparent power,  $S$ . Apparent power is discussed, in greater detail, in Segment 2. Practical significance of the ideal transformer equations is illustrated through Example 1.4

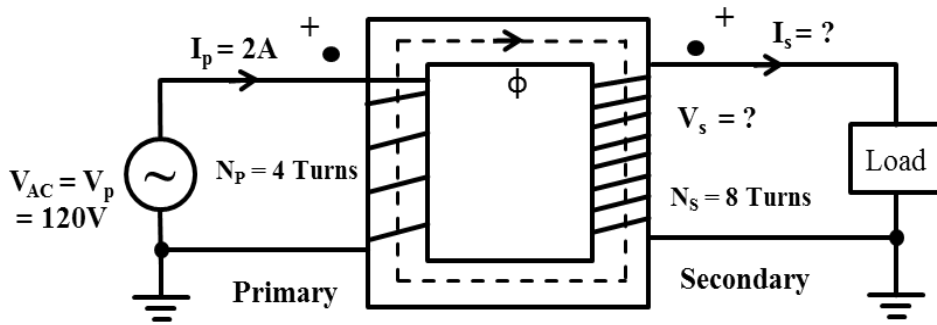
#### **Example 1.4**

The primary of the transformer shown in Figure 1.9 is fed from a 120 VAC source. As apparent from the circuit diagram, the transformer has four (4) turns on the primary and eight (8) turns on the secondary. The current measured on the primary, using a clamp-on ammeter, is 2A. Determine the following unknown parameters:

- (a) Turns ratio
- (b) Secondary current,  $I_s$
- (c) Secondary voltage,  $V_s$

#### **Solution:**

The scenario described in this example can be illustrated pictorially by annotating the circuit diagram Figure 1.9 as follows:



(a) According to ideal transformer equation Eq. 1.12,

As given,  $N_p = 4$  and  $N_s = 8$ .

$$\therefore \text{Turns Ratio} = \left( \frac{4}{8} \right) = \frac{1}{2} \text{ or } 1:2$$

(b) According to ideal transformer equation Eq. 1.14,

$$I_s = a \cdot I_p = \left( \frac{N_p}{N_s} \right) \cdot I_p$$

Since the turns ratio was computed as  $\frac{1}{2}$  in part (a), and primary current  $I_p$  is given as 2A in the problem statement,

$$I_s = \left( \frac{1}{2} \right) \cdot I_p = \left( \frac{1}{2} \right) \cdot (2A) = 1A$$

(c) According to ideal transformer equation Eq. 1.13,

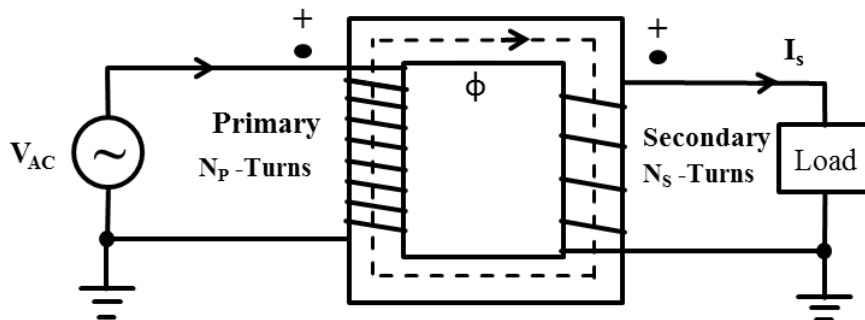
$$V_s = \left( \frac{V_p}{a} \right)$$

Since the turns ratio was computed as 1:2 or  $\frac{1}{2}$  in part (a), and primary voltage  $V_p$  is given as 120V in the problem statement,

$$V_s = \left( \frac{V_p}{a} \right) = \left( \frac{120V}{1/2} \right) = 240V$$

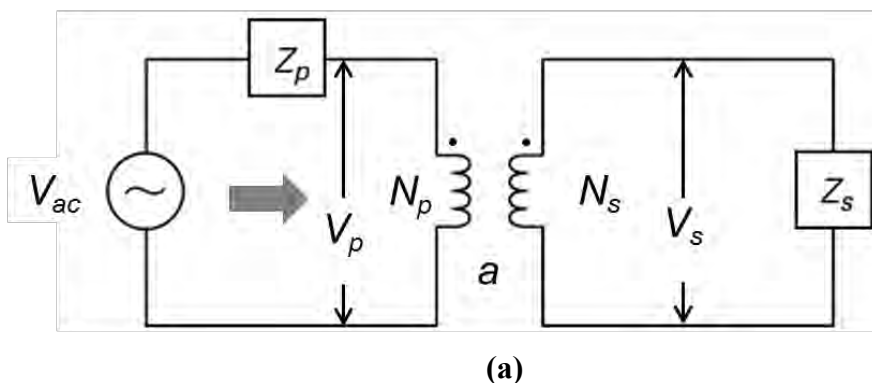
In Example 1.4, the number of secondary turns,  $N_s$ , is twice the number of primary turns,  $N_p$ ; resulting in the secondary voltage that is twice the magnitude of the primary voltage. This configuration is called a “*step up*”

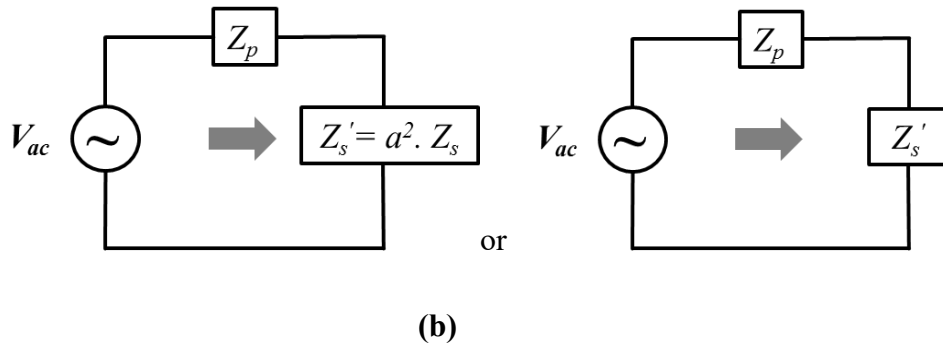
transformer configuration. In other words, the transformer is being used to boost or step up the primary voltage. Conversely, as shown in Figure 1.11, if the primary windings are greater than the secondary windings, the transformer would constitute a “step down” transformer. Such configuration is used to reduce incoming utility or source voltage to a desired lower voltage that is compatible with the driven load or equipment. See self-assessment problem 4, at the end of this segment, for a practical illustration of a step down transformer scenario and associated analysis.



**Figure 1.11:** A Step down transformer configuration

The ideal transformer model shown in Figure 1.11 can be expanded to include impedance on the primary side of the transformer and the impedance of the load. If those two impedances are introduced into the discussion, as is the case in practical AC electrical systems, the AC transformer circuit would be represented by the circuits shown in Figures 1.12 (a) and (b).





**Figure 1.12:** (a) Ideal transformer model with impedance consideration (b) Alternate representations of an ideal transformer equivalent circuit.

Note that the left to right arrows denote the flow of AC power, from the source to the load. The explanation of the nomenclature used in Figure 1.12 (a) and (b) is as follows:

$V_{ac}$  = AC voltage source

$V_p$  = Voltage on the primary side of the transformer

$V_s$  = Voltage on the secondary side of the transformer

$a$  = Turns ratio =  $N_p/N_s$

$Z_p$  = Impedance between the AC voltage source and the primary coil of the transformer.

$Z_s$  = Impedance of the load connected to the secondary side of the transformer.

$Z'_s$  = Impedance of the load as seen from the primary side of the transformer =  $a^2 \cdot Z_s$

Impedances  $Z_p$  and  $Z_s$  do not represent the impedances of the transformer itself. In other words,  $Z_p$  represents the combined impedance of all circuit elements connected to the primary side of the transformer and  $Z_s$  represents the load impedance or the combined impedance of all circuit elements connected to the secondary side of the transformer. Simplification of the transformer circuit involved in the transition from Figure 1.12 (a) to Figure 1.12 (b) can be followed through the basic steps listed below:

- 1) Combine the impedances of all circuit elements connected between the primary side of the transformer and the AC voltage source. This combined impedance is represented by  $Z_p$ .

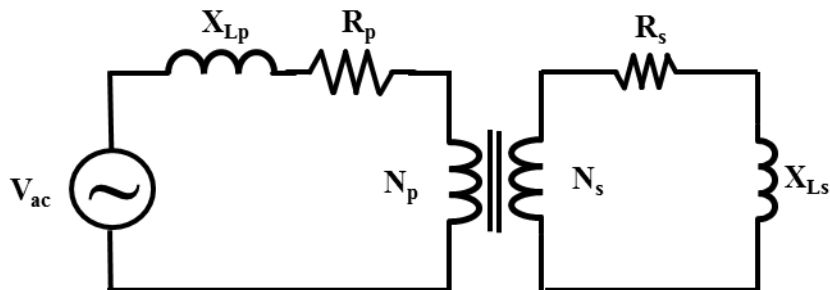
- 2) Combine the impedances of all circuit elements connected between the secondary side of the transformer and the load. This combined impedance is represented by  $Z_s$ .
- 3) The combined secondary impedance  $Z_s$  is reflected to the primary side - or appears from the primary side of transformer - as  $Z'_s$ . The symbol  $Z'_s$  is read as “**Z**” sub “**s**” *prime*. The relationship between  $Z_s$ ,  $Z'_s$  and the turns ratio  $a$  is represented by the Eq. 1.16 below:

$$Z'_s = a^2 \cdot Z_s \qquad \text{Eq. 1.16}$$

- 4) The original transformer circuit with numerous elements can then be shown in a simplified or condensed form – with only three elements:  $V_{ac}$ ,  $Z_p$  and  $Z'_s$  – as shown in Figure 1.12 (b).

**Example 1.5**

Calculate the equivalent impedance as seen from the vantage point of the AC source  $V_{ac}$  in the circuit shown below. The transformer in the circuit is assumed to be ideal. The values of the primary and secondary circuit elements are:  $X_{Lp} = 2\Omega$ ,  $R_p = 4\Omega$ ,  $R_s = 10\Omega$ ,  $X_{Ls} = 5\Omega$ ,  $N_p = 100$ ,  $N_s = 200$ .



**Solution:**

Note that, for the sake of simplicity, the primary and secondary inductors are given in form of their reactances “ $X_L$ ” instead of actual inductance values, in form of Henries. As explained earlier in this segment, in the process of reducing the AC circuits to an equivalent impedance form, the individual inductive and capacitive reactances are combined in form of their respective impedance, or  $Z$ , contributions. In other words,  $X_L$ ’s are represented by  $Z_L$ ’s and  $X_C$ ’s are represented by  $Z_C$ ’s.

Employing steps (1) through (4) from the section above results in the following computation and the equivalent circuit:

On the primary side:  $\mathbf{Z}_{Lp} = jX_{Lp} = j2\Omega$   
 Therefore,  $\mathbf{Z}_p = R_p + \mathbf{Z}_{Lp} = 4 + j2\Omega$

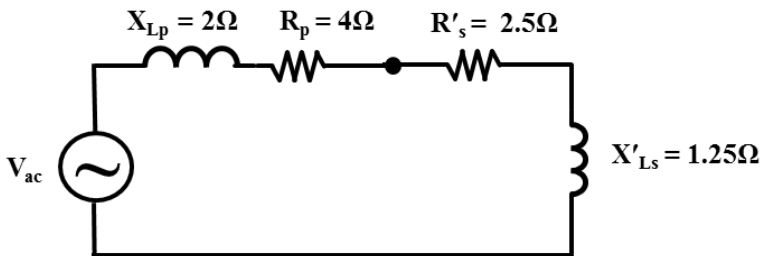
On the secondary side:  $\mathbf{Z}_{Ls} = jX_{Ls} = j5\Omega$   
 Therefore,  $\mathbf{Z}_s = R_s + \mathbf{Z}_{Ls} = 10 + j5\Omega$

$a = \text{Turns ratio} = N_p/N_s = 100/200 = 1/2$

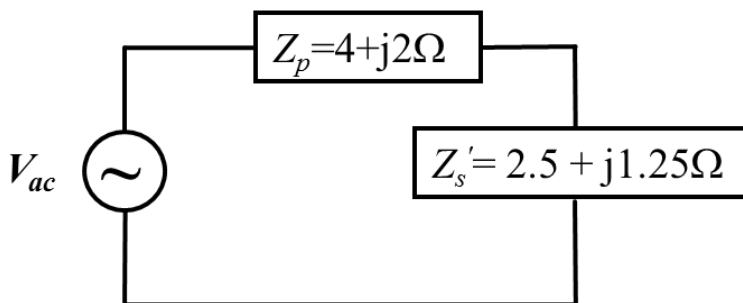
$$\mathbf{Z}'_s = a^2 \cdot \mathbf{Z}_s$$

$$\mathbf{Z}'_s = (1/2)^2 \cdot (10 + j5\Omega) = 2.5 + j1.25\Omega$$

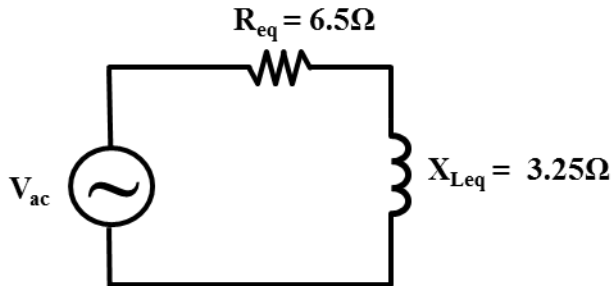
$$\mathbf{Z}_{eq} = \mathbf{Z}_p + \mathbf{Z}'_s = (4 + j2\Omega) + (2.5 + j1.25\Omega) = \mathbf{6.5 + j3.25\Omega}$$



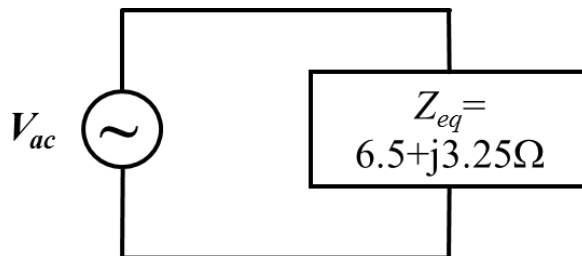
Or,



And, combination of the two impedances, as supported by the math above in:



The final equivalent impedance  $Z_{eq}$  can be represented in rectangular complex form as:



The  $Z_{eq}$  derived in the rectangular for above can be stated in polar or phasor form as:

$$Z_{eq} = 6.5 + j3.25\Omega = 7.27 \angle 26.6^\circ$$

This conversion from rectangular to phasor form can be accomplished through a scientific calculator, with complex math feature or, as illustrated earlier in this segment, through application of Pythagorean Theorem and trigonometry.

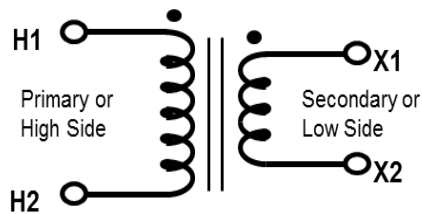
## Common Single Phase AC Transformer Applications

Three common single phase transformer configurations are described in this section, accompanied by their respective schematics.

**Generic step-up or step-down single phase transformer application:** Even though the single phase transformer schematic shown in Figure 1.13 represents greater number of turns on the primary side versus the secondary side – implying that it is a step down transformer - this transformer could be



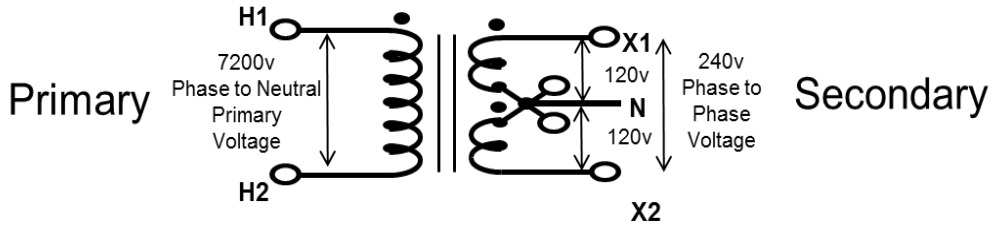
wired or connected in a reverse fashion, resulting in a step up voltage transformation function.



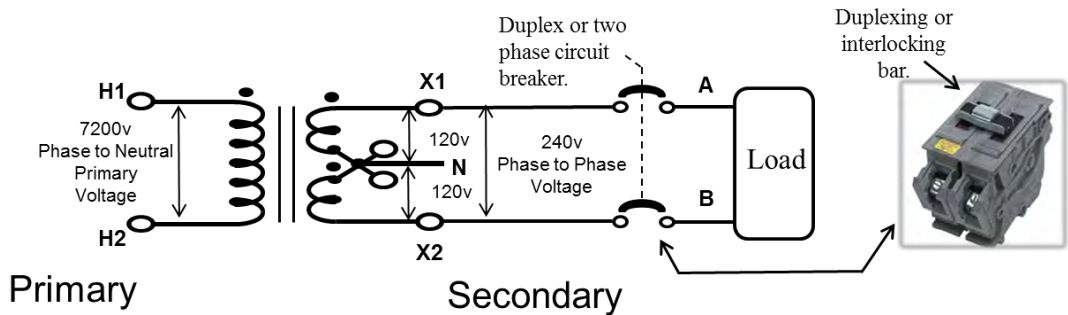
**Figure 1.13:** Generic step-down single phase transformer configuration

**Step down voltage transformation - residential power distribution:** Single phase transformer schematic shown in Figure 1.14 represents step down voltage transformation typically utilized in residential power distribution applications. As shown in the schematic, typically, 7200 volts - in phase ( $\phi$ ) to neutral three phase Y power supply configuration - are fed to the primary of a single phase power transformer. The secondary of the residential power distribution transformer is split or “tapped” into two segments, with the center tap connected to a neutral or ground. Such secondary configuration allows electricians to utilize both 120 volt,  $\phi$  – neutral, segments for multiple 120 volt loads at a particular residence. On the other hand, for the 240 volt loads, such as clothes dryers, water heaters, HVAC systems and electric ranges, the electricians connect the line side of the respective load breakers across terminals  $X_1$  and  $X_2$ , phase to phase.

As shown in Figure 1.14, the 7200 volts presented to the primary of this residential power distribution transformer are stepped down to the secondary voltage of 240 volts. Both terminals  $X_1$  and  $X_2$  are energized or “hot.” That is why, if you examine a typical residential breaker panel, you will note that the breakers for loads such as HVAC systems, dryers, water heaters and ranges are installed in duplex or dual formation. In other words, two breakers are connected per circuit; one on each energized phase. See the breaker interlocking bar - mechanically interconnecting the two breakers together for simultaneous operation - in Figure 1.15. This interlocking or duplexing feature permits simultaneous breaking/operation of both energized phases when the breaker trips or is turned off. The schematic segment of Figure 1.15 shows the electrical interface and location of the breakers in two phase systems.



**Figures 1.14:** Step down, residential power distribution transformer

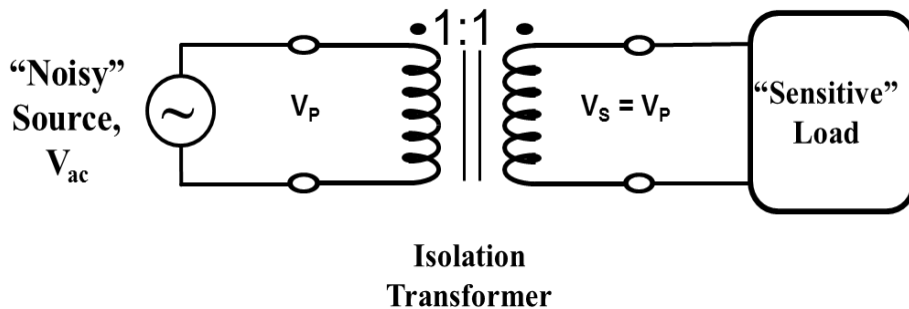


**Figures 1.15:** Step down, residential power distribution transformer with load

**Load to source, power isolation transformer:** In most industrial and many commercial facilities, equipment such as arc welders, intermittent motor loads and other types of switching loads tend to feedback electric “spikes” or “noise” to the supply line or supply bus. Such undesirable noise and spikes on the supply line or feeder are subsequently seen by all loads sharing the same supply line. Such switching of electromagnetic circuits also results in generation of electromagnetic radiation. This electromagnetic radiation, sometimes referred to as EMI, or electromagnetic interference, can have an adverse effect on sensitive electronic equipment such as computers, TV’s, audio/video stereo systems, VFD’s or variable frequency drives, PLC’s (Programmable Logic Controllers), PC’s (Personal Computers) and other types of electronic equipment. The electrical noise on the utility or power supply lines and EMI often tend to cause trips, faults and computational errors in sensitive electronic equipment.

There are various methods for attenuating or eliminating the electrical noise on the power supply lines and power distribution systems. Some such approaches include the application of capacitors and electrical filters that serve as “sponges” or noise absorbers. Another common method for

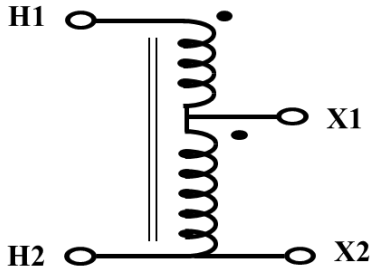
protecting against electrical noise consists of an isolation transformer. See Figure 1.16 for a schematic of a typical, single phase isolation transformer. A distinguishing characteristic of an isolation transformer is that its primary to secondary winding turns ratio is 1:1. In other words, in isolation transformers, the number of turns on the primary side equals the number of turns on the secondary side. So, the primary and secondary voltages are the same. The net gain is the “cleansing” of voltage or current achieved through conversion electrical current to magnetic flux  $\phi$  on the primary side and subsequent conversion of the flux to electrical AC current in the load or secondary side of the transformer. This flux to current and current to flux conversion are depicted in Figure 1.11.



**Figures 1.16:** Load to source, 1:1, power isolation transformer.

### **Autotransformers**

An autotransformer is a type of transformer that requires fewer windings and a smaller core. Therefore, it offers an economical and lighter means for converting voltage from one level to another as compared with its, regular, two winding counterparts. Practical, cost effective, application of an autotransformer is typically limited to a voltage ratio of about 3:1.



**Figure 1.17:** Autotransformer

A typical autotransformer has one winding and four electrical connections, referred to as taps. In autotransformers, a portion of the winding serves as the primary as well as the secondary. See Figure 1.17 where the winding spanning from terminals  $X_1$  to  $X_2$  serves as the primary as well as the secondary.

## Voltage Regulation, Voltage Regulators and Buck Boost Transformers

Envision a scenario where you own and manage a small factory that operates on an 8 to 5 day shift. Assume that all power is turned off at the main breaker at the end of the day. As the first person to arrive at the facility in the morning, in an effort to assess the power quality of your facility, you measure the voltage at the main breaker before any electrical equipment is turned on. Let's assume that the voltage you measure is  $480 V_{RMS}$ . If you measure the voltage at that main breaker, once again, after turning on all of the routine load, as well as, your production machinery, you will most likely read some lower voltage, say  $460 V_{RMS}$ . This reduction in voltage, as load is added to the power distribution system, when quantified as a percentage, is referred to as voltage regulation. As the load on a power distribution system is increased, the current being pulled or demanded, from the source, by the load, increases. Since the ultimate mechanical source of electrical energy has definite or limited capacity, or limited and relatively constant power,  $P$ , as the demand for current  $I$  rises, the voltage sags proportionally. This can be explained on the basis of Eq. 1.17 and 1.18.

For DC systems,  $P = V.I$ , or,

$$I = \frac{P}{V} \text{ or, } V = \frac{P}{I} \quad \text{Eq. 1.17}$$

In AC systems, **S**, AC Apparent Power = **V.I**, or,

$$|I| = \frac{|S|}{|V|} = |V| = \frac{|S|}{|I|} \quad \text{Eq. 1.18}$$

As more current is demanded by loads, more current flows through DC supplies and AC power transformers. This results in a voltage drop on the load side of the DC power supply or AC power transformer, as the load approaches the maximum power capacity of the system. For some voltage, regulation can be best understood and applied in its mathematical form as stated in Eq. 1.19.

$$\text{Voltage Regulation in \%} = \left[ \frac{V_{NL} - V_{FL}}{V_{FL}} \right] \times 100 \quad \text{Eq. 1.19}$$

Where,

$V_{NL}$  = voltage measured at the source under no load condition

$V_{FL}$  = voltage measured at the source under full load condition

Note that Eq. 1.19 may appear in a slightly different form in some texts, as stated in Eq. 1.20.

$$\text{Voltage Regulation in \%} = \left[ \frac{V_{NL} - V_{Rated}}{V_{Rated}} \right] \times 100 \quad \text{Eq. 1.20}$$

In Eq. 1.20,  $V_{Rated}$  is considered synonymous with the full load voltage,  $V_{FL}$ . In other words, the rated voltage is assumed to be the voltage when the system is operating at full load.

As apparent from Eq. 1.19, in an ideal “utopian” scenario, voltage regulation would approach “0.” In other words, in an ideal situation, the voltage magnitude at the source would stay constant as the load escalates from no load to rated or full load level. A source that offers a low or negligible regulation is sometimes referred to, by electricians or electrical engineers, as a “*stiff*” source. In a typical power distribution system, a voltage regulation of

3' 'y qwf 'dg'eqpuf gtgf 'cu'ceegr vcdrg'qxgt 'kpr w'xqnci g'xctkvwqpu'qh'- 32' " vq"/42' 0'Xqnci g'tgi wrcvqp'vqpf u'vq'dg'rti gt '\*qt'y qtug+'kp'kpf wvwxg'mqcf " u{vgo u'cu'eqo r ctgf 'y kj 'tgukvwxg'mqcf 'u{vgo u0'Rqy gt 'f kvtkdwkqp'u{vgo u" y kj 'rti gt 'xqnci g'tgi wrcvqp'ctg'uqo g'wko gu'tghgtgf 'vq'cu'gzj kdkkpi '-looseø' xqnci g'tgi wrcvqp0'

**Gzco riq'3Ø"**

Cuuwo g'vj cv'vj g'ugeqpf '\*tvcgf 'qt'hwmlmqcf '+xqnci g'o gcuwtgf 'kp'vj g'xqnci g" tgi wrcvqp'uegpctkq'ecr wtgf 'kp'vj g'htuv'r ctcj tcr j 'qh'vj ku'ugevqp'ku'682'X<sub>TOU0'</sub> F gvgto kpg'vj g'xqnci g'tgi wrcvqp0'

**Uqnvkqp<'**

**X<sub>PN</sub>**? '6: 2'X.'i kxgp"

**X<sub>FN</sub>**? '682'X.'i kxgp"

Cr r nq'Gs03Q <"

$$\text{Voltage Regulation in \%} = \left[ \frac{V_{NL} - V_{FL}}{V_{FL}} \right] \times 100$$

$$\text{Voltage Regulation in \%} = \left[ \frac{480 - 460}{460} \right] \times 100 = 4.35\%$$

""""Nqqug'qt'wpceegr vcdrg'xqnci g'tgi wrcvqp'ecp'dg'tgo gf kqf 'vj tqwi j " xctkqwu'o gcpu.'vj tgg'qh'uwej 'o gyj qf u'ctg'hwkqf 'dgmjy <"

3+Cr r nrcvqp'qh'dwem/dqqv'xqnci g'tgi wrcvqi 'tcpuhtqto gtu

4+Cr r nrcvqp'qh'xqnci g'tgi wrcvqi 'tcpuhtqto gtu

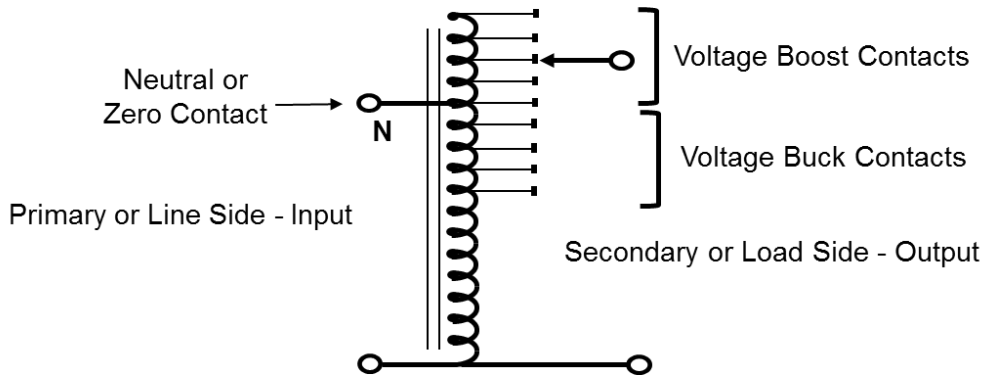
5+Cr r nrcvqp'qh'htgtqtguqpcpv.'o ci pgvle'ucwrcvqp'dcuqf .'tcpuhtqto gtu

Y g'y kn'grcdqtcvg'qp'vj g'o qtg'eqo o qp'dwem/dqqv'xqnci g'tgi wrcvqi tcpuhtqto gt'kp'vj ku'vgz0C'uej go c'vle'f kci tco 'qh'c'dwem/dqqv'xqnci g" tgi wrcvqi 'tcpuhtqto gt'ku'uj qy p'kp'Hi wtg'3Ø: 0Vj ku'v'r'qh'xqnci g'tgi wrcvt" ku'dwkn'ctqwpf 'cp'cwq/tcpuhtqto gt0Kj'cu'e'o qxcdrg'eqpvcv'vj cv'o cngu" gngvteknleqppgevqp'y kj 'cxckrcdrg'eqki'vr u0"

""""C'dwem/dqqv'cwqvtcpuhtqto gt'v'r g'xqnci g'tgi wrcvt'ecp'tckug'xqnci g"

\*dqqv'qt'hwj gt 'xqnci g'dwem'cu'pgeguact {0Vj gug'tgi wrcvtu'ctg.'v'r kcmf ."

rated +/- 10%; meaning that they are capable of maintaining the output (or secondary) voltage within +/- 1% in response to +/- 10% variation of the input (or primary) voltage.



**Figures 1.18:** Buck-Boost Voltage Regulating Autotransformers

### Three Phase AC

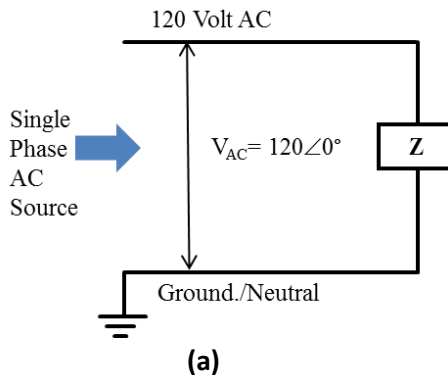
While single phase AC is based on one energized phase (or conductor) and a ground or neutral, three phase AC systems consist of three energized phases (or conductors). Some three phase AC systems consist of three energized conductors and a grounded neutral. Note that electrical engineers and electricians often refer to the energized conductors as “*hot*” conductors. When measuring voltages in single phase or three phase systems, the energized or “hot” terminal or conductor is touched with the red (anode) probe of the voltmeter and the black (cathode) probe is connected to the neutral or ground terminal. See Figure 1.19 below for a contrast between a single phase system and a three phase system. The single phase circuit, shown in Figure 1.19 (a), is powered by a 120 volt AC source. This single phase AC source is assumed to have an angle of  $0^\circ$ . Therefore, complete vector or phasor representation of the source voltage would be  $120\angle 0^\circ$  VAC. The load in this single phase AC circuit is represented by impedance “*Z*.”

On the other hand, Figure 1.19 (b) represents a three phase AC circuit, with a three phase AC source supplying power to three phase AC load. Note that the three phases in the three phase AC system are identified as phases A, B and C. The three phase load consists of: (a) impedance  $Z_A$ , connected between phase A and the neutral, (b) impedance  $Z_B$ , connected between phase B and the neutral, and (c) impedance  $Z_C$ , connected between phase C and the

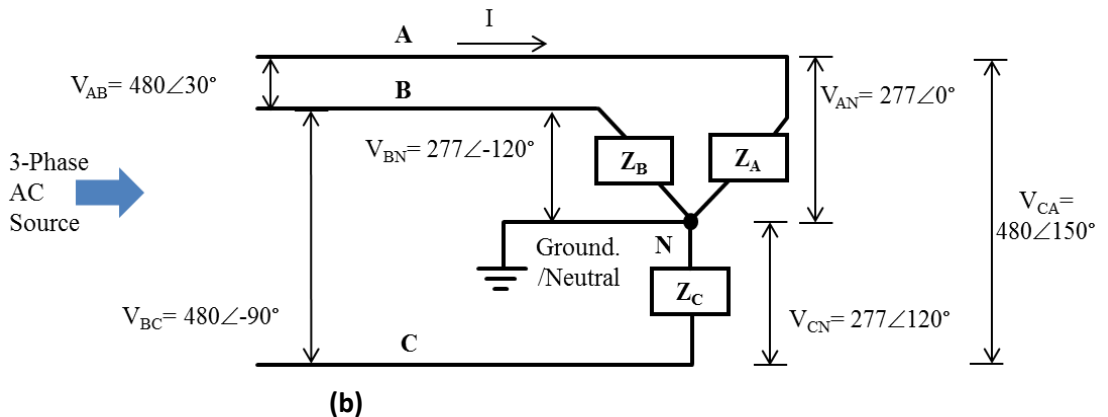
neutral. In many common three phase AC load scenarios - such as three phase motors – the impedance values for each of the three phases are equal. When the three phases present equal impedance, the three phase load is said to be balanced. In other words:

$$Z_A = Z_B = Z_C = Z$$

Unequal phase impedances result in an imbalanced load condition, unequal line currents, and in some cases, unequal line voltages. Under most normal circumstances, the neutral current  $I_N$  in Y-loads is negligible. However, under unbalanced load conditions,  $I_N$  is non-zero. Therefore, neutral line current  $I_N$  is often monitored by installing a current transformer (CT) on the neutral leg of the three phase system. See additional discussion on current transformers and their application in the next section. Due to the fact that the line fuses or breakers protecting the three phase systems are sized typically with the assumption of balanced phase loading, when loads phases become unbalanced, currents in one or more of the three phases can exceed the fuse or breaker trip threshold; thus, resulting in the clearing or “blowing” of the fuse or tripping of the breaker.







**Figure 1.19:** (a) Single phase AC system. (b) Three –Phase AC System, Y-Configuration.

### Three Phase AC Systems vs. Single Phase AC Systems

The advantages of three phase AC systems over their single phase AC counterparts are as follows:

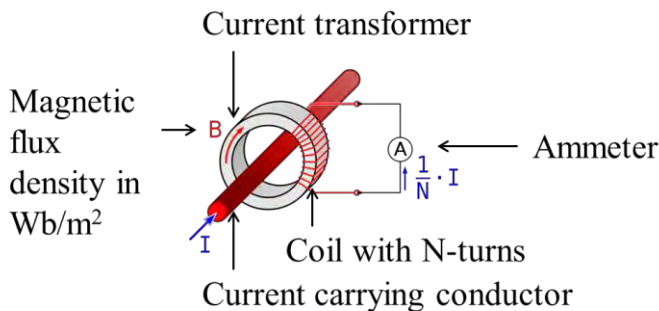
- a) Three phase circuits and power distribution systems are more efficient than single phase systems. In other words, three phase power distribution systems can deliver the same magnitude of power with fewer and smaller conductors.
- b) Unlike single phase induction motors, three phase AC induction motors do not require additional starting windings.
- c) Unlike single phase induction motors, three phase AC induction motors provide uniform torque. Single phase motors often tend to deliver pulsating torque.
- d) When rectifying AC current to DC current, three phase AC rectification yields smoother, relatively ripple free, DC.

### Current Transformers

Current transformers are transformers that are used to measure or sample currents in AC electrical circuits. In measurement applications, a current transformer, abbreviated as “CT,” is connected to metering instrumentation to

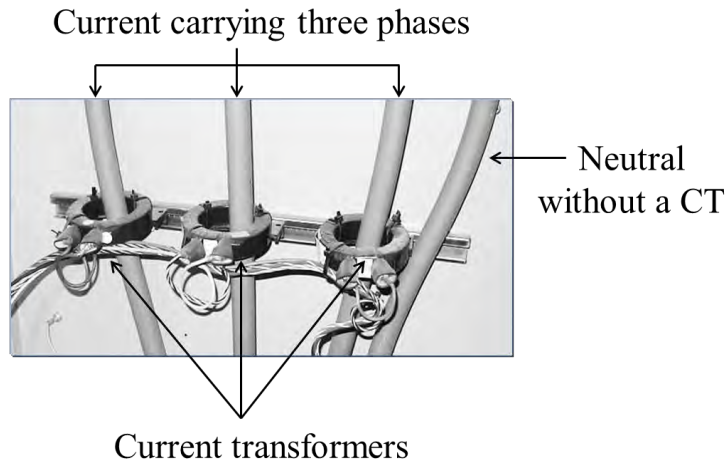
display the magnitude of AC current through an AC ammeter or some other form of annunciation, such as a computer based HMI, Human Machine Interface, system. As shown in Figure 1.20 below, the conductor whose current is to be measured, is routed through the CT. When current,  $I$ , flows through the conductor, it produces magnetic flux – with magnetic flux density  $B$  – in the cylindrical core of the CT. This magnetic flux, as explained in Segment 2, initiates the flow of current through the windings of the CT. The current  $I_{CT}$  induced in the windings of the CT is a proportional representation of the current  $I$  flowing through the conductor being monitored. The proportional relationship between  $I$  and  $I_{CT}$  can be represented, mathematically, by Eq. 1.21.

$$I_{CT} = \frac{I}{N} \quad \text{Eq. 1.21}$$



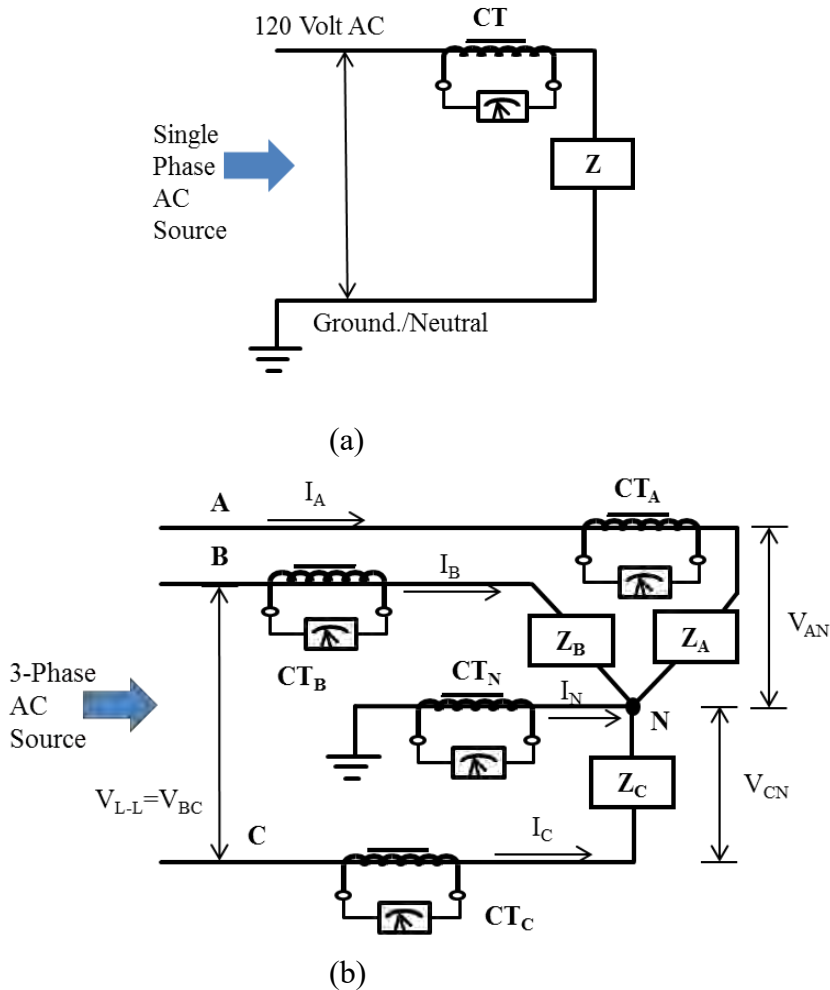
**Figure 1.20:** Current transformer operating principle. By: Beizl, April 15, 2009.

When CT's are used to measure phase or line currents in three phase AC systems, they are installed in a fashion depicted in Figure 1.21. The CT installation photographed and shown in Figure 1.21 pertains to a situation where only the three energized phases need to be metered; a CT on the neutral is, therefore, not needed.



**Figure 1.21:** Current transformers used as part of metering equipment for three phase 400Amp electricity. By: Ali, December 13, 2004.

While majority of CT applications pertain to metering or display of AC current, in form or another, CT's are often applied for control purposes. When CT's are included in electrical power distribution systems for control purposes, they are terminated at – or connected to – the Input/Output (I/O) blocks of a control system, such as, a Programmable Logic Controller (PLC) or a Direct Digital Control (DDC) System. In such control applications, the current induced in the CT windings is often scaled or transduced to a low voltage signal at the I/O blocks. In many control systems, the PLC or DDC's CPU (Central Processing Unit) make control decisions on the basis of the scaled signal, available at the CT I/O block, by continuous monitoring and comparison of the real time current signal against “**High**” or “**Low**” set points. These “**High**” or “**Low**” set points are embedded in the program or algorithm of the PLC or DDC; where, control programs typically cycle at the rate of approximately once per 20 milliseconds. The scan rates are a function of the complexity of the program and the number of inputs and outputs included in the architecture of the overall control system. Tripping of breakers, shutting down of equipment, turning on of alarms and horns are a few examples of controls functions executed on the basis of current sensed by CT's. Schematics in Figures 1.22 (a) and (b) show the distinction between CT's being applied in single phase AC scenarios versus three phase AC scenarios.



**Figure 1.22:** (a) Current transformer applied to a single phase AC circuit, for metering purposes. (b) Current transformers applied to a three phase AC circuit, for metering purposes.

### Three Phase AC Transformers

Three phase transformers, like the basic single phase transformers, serve the purpose of transforming voltage from one magnitude to another. Like single phase transformers, three phase transformers can be used as isolation transformers, where the primary and secondary voltages are the same and the main purpose of the transformer is to insulate and isolate a sensitive load from electrical noise and voltage fluctuations prevalent on the AC supply line. Three phase transformers' intrinsic and operational parameters, such as impedance, voltage and current – similar to their single phase counterparts –

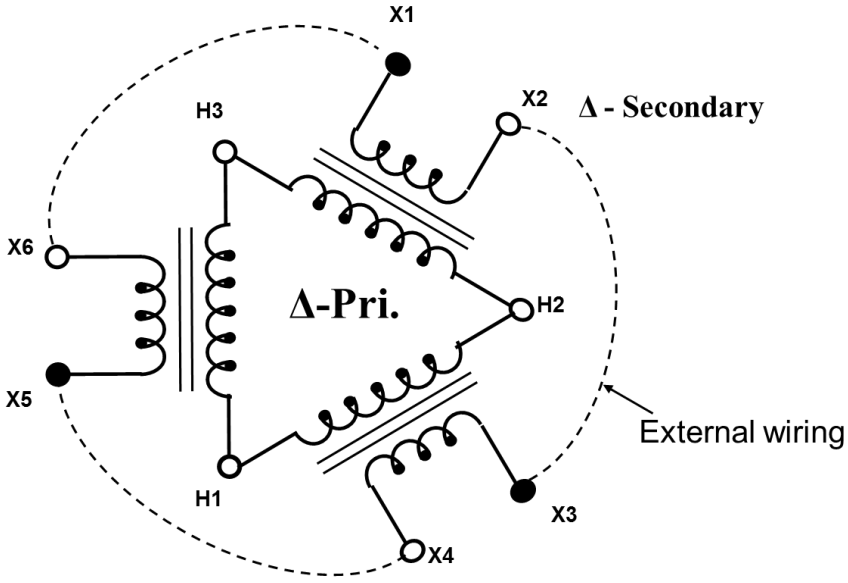
are represented by complex numbers and vectors. Upon close examination of Figure 1.19 (b) it is apparent that in three phase transformers, the voltages and currents for each of the three phases are separated by  $120^\circ$ .

Three identical single-phase transformers can be connected to form a 3-phase bank. Primary and secondary sides of such a bank of three phase transformers can be connected in Star (Y) – Delta ( $\Delta$ ), Star (Y) – Star (Y) or Delta ( $\Delta$ ) – Delta ( $\Delta$ ) combinations. Various possible three phase transformer configurations are shown in Figures 1.23 through 1.30.

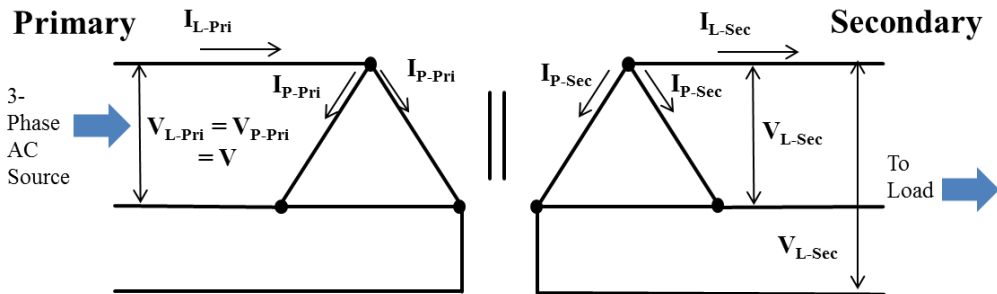
When determining the voltages and currents in a 3-phase transformer, one must employ the line and phase relationship of star or delta connections along with the ratio of transformation between the coupled windings as explained in the various transformer configurations below.

### **Three Phase $\Delta$ - $\Delta$ Transformer Configuration**

Most transformers, unless otherwise specified, are delivered to customers or installation sites without final terminations or “pre-configuration;” meaning, unless otherwise specified, they are not prewired for  $\Delta$ - $\Delta$ ,  $\Delta$ -Y, Y- $\Delta$ , or Y-Y application. One could view an “un-configured” three phase transformer as a combination of three single phase transformers. As such, a set of three individual single phase transformers could be wired or configured as  $\Delta$ - $\Delta$ ,  $\Delta$ -Y, Y- $\Delta$ , or Y-Y. If three individual transformers are connected together as depicted in Figure 1.23 (a), the resulting configuration would be a  $\Delta$ - $\Delta$  three phase transformer, where the points labeled X1 through X6 and H1 through H3 are terminals, wire termination points, or physical connection points. The schematic version of this  $\Delta$ - $\Delta$  transformer would be represented by Figure 1.23 (b). The schematic, as usual, conveys the functional information about the transformer. In other words, the schematic illustrates how the voltage and current get transformed between the primary and secondary of the transformer.



(a)



(b)

**Figure 1.23:**  $\Delta-\Delta$ , or Delta – Delta, 3–Phase Transformer Configuration. (a) Wiring diagram, (b) Schematic

As shown in Figure 1.23 (b), key mathematical relationships between the phase voltages and line voltages, and phase currents and line currents are as follows:

$$V_{P-Pri} = V_{L-Pri} = V \quad \text{Eq. 1.22}$$

$$V_{P-Sec} = V_{L-Sec} \quad \text{Eq. 1.23}$$

$$I_{P-Pri} = \frac{I_{L-Pri}}{\sqrt{3}} = \frac{I}{\sqrt{3}} \quad \text{Eq. 1.24}$$

$$I_{L-Sec} = \sqrt{3} \cdot I_{P-Sec} = a \cdot I \quad \text{Eq. 1.25}$$

Where,

$V_{P-Pri}$  = Primary phase voltage =  $V$

$V_{L-Pri}$  = Primary line voltage =  $V$

$V_{P-Sec}$  = Secondary phase voltage

$V_{L-Sec}$  = Secondary line voltage

$I$  = Primary line current

$I_{P-Pri}$  = Primary phase current

$I_{P-Sec}$  = Secondary phase current

$I_{L-Pri}$  = Primary line current =  $I$

$I_{L-Sec}$  = Secondary line current

For the sake of simplicity, and in congruence with the context and scope of this text, we will limit the discussion in this three phase transformer section to the magnitudes of currents and voltages. Of course, when full vector or phasor analyses of voltages and currents are required, the angles of voltages and currents must be considered in conjunction with the respective magnitudes.

As illustrated in Figure 1.24, the line voltage and line current transformations for  $\Delta$ - $\Delta$  three phase transformers are governed by the following equations:

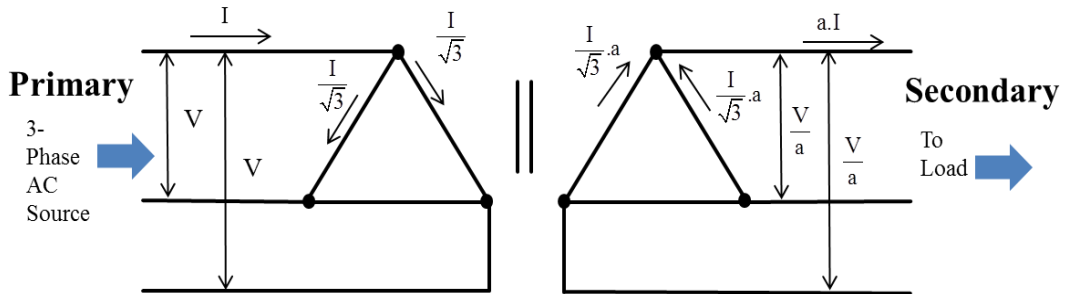
$$\frac{V_{L-Sec}}{V_{L-Pri}} = \frac{1}{a} \quad \text{Eq. 1.26}$$

or,  $V_{L-Sec} = \frac{V_{L-Pri}}{a} = \frac{V}{a}$

$$\frac{I_{L-Sec}}{I_{L-Pri}} = a$$

Eq. 1.27

or,  $I_{L-Sec} = a \cdot I_{L-Pri} = a \cdot I$



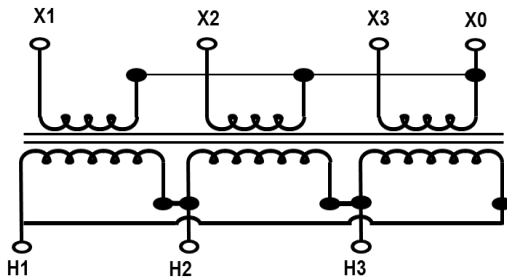
**Figure 1.24:**  $\Delta - \Delta$ , or Delta – Delta, voltage and current transformation

### Three Phase $\Delta$ -Y Transformer Configuration

The wiring diagram for a three phase  $\Delta$ -Y transformer is depicted in Figure 1.25(a). In this diagram, terminals H1 through H3 represent the primary “ $\Delta$ ” connection points and terminals X0 through X3 represent the secondary “Y” connection points. In the wiring diagram, in accordance with electrical convention, the solid, circular, black dot represents an internal connection that determines the  $\Delta$  or Y configuration of the transformer primary or secondary. The wires connecting the solid black dot terminals are, sometimes, referred to as “jumpers.” The hollow circular rings represent the terminals where external field connections are terminated. The schematic version of this  $\Delta$ -Y transformer is shown in Figure 1.25 (b).

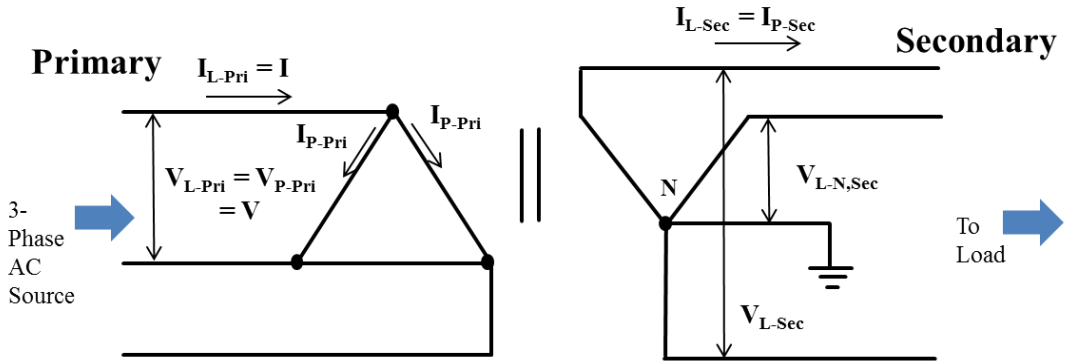


Y - Secondary



$\Delta$  - Primary

(a)



(b)

**Figure 1.25:** (a)  $\Delta$ –Y, or Delta – Wye, transformer wiring diagram, (b)  $\Delta$ –Y, transformer schematic

As illustrated in Figure 1.25 (b), key mathematical relationships between the phase voltages and line voltages, and phase currents and line currents are as follows:

$$V_{P-Pri} = V_{L-Pri} \quad \text{Eq. 1.28}$$

$$V_{L-Sec} = V_{P-Sec} \cdot \sqrt{3} \quad \text{Eq. 1.29}$$

$$I_{P-Pri} = \frac{I_{L-Pri}}{\sqrt{3}} = \frac{I}{\sqrt{3}} \quad \text{Eq. 1.30}$$

$$I_{P-Sec} = I_{L-Sec} \quad \text{Eq. 1.31}$$

Where,

- $V_{P-Pri}$  = Primary phase voltage =  $V$
- $V_{L-Pri}$  = Primary line voltage =  $V_{Line-Delta} = \sqrt{3}V$
- $V_{P-Sec}$  = Secondary phase voltage
- $V_{L-Sec}$  = Secondary line voltage =  $V_{Line-Y}$
  
- $I_{L-Pri}$  = Primary line current =  $I$
- $I_{P-Pri}$  = Primary phase current
- $I_{P-Sec}$  = Secondary phase current =  $I_{Line-Y}$
- $I_{L-Sec}$  = Secondary line current =  $I_{Line-Y}$

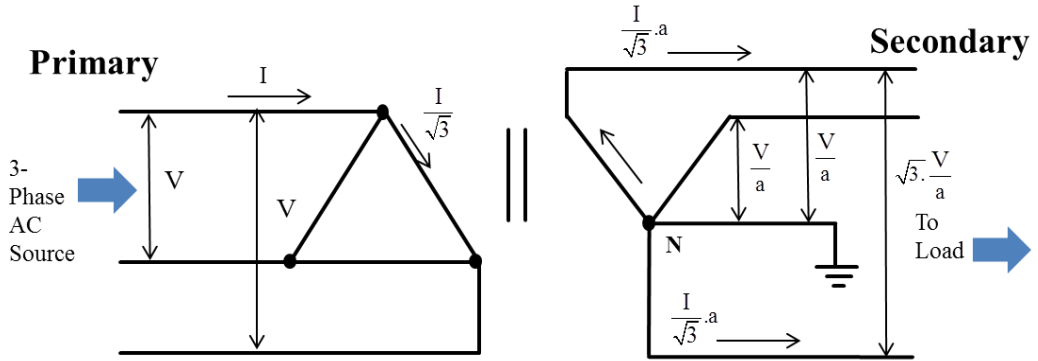
For a  $\Delta$ -Y three phase transformers, as illustrated in Figure 1.26, the voltage and current transformations can be assessed using the following equations:

$$\frac{V_{Line-Y}}{V_{Line-Delta}} = \frac{\sqrt{3}}{3}$$

or,  $V_{Line-Y} = \frac{\sqrt{3}}{3} \cdot V_{Line-Delta} = \frac{\sqrt{3}}{3} \cdot \sqrt{3}V$  Eq. 1.32

$$\frac{I_{Line-Y}}{I_{Line-Delta}} = \frac{1}{\sqrt{3}}$$

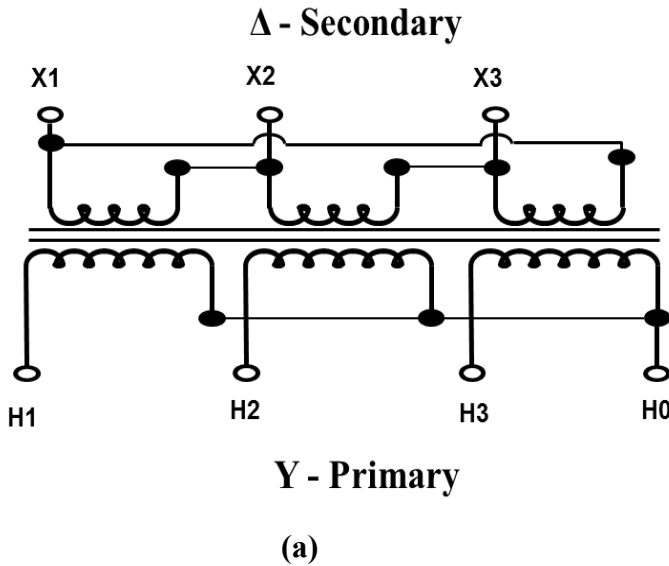
or,  $I_{Line-Y} = \frac{1}{\sqrt{3}} \cdot I_{Line-Delta} = \frac{1}{\sqrt{3}} \cdot I$  Eq. 1.33

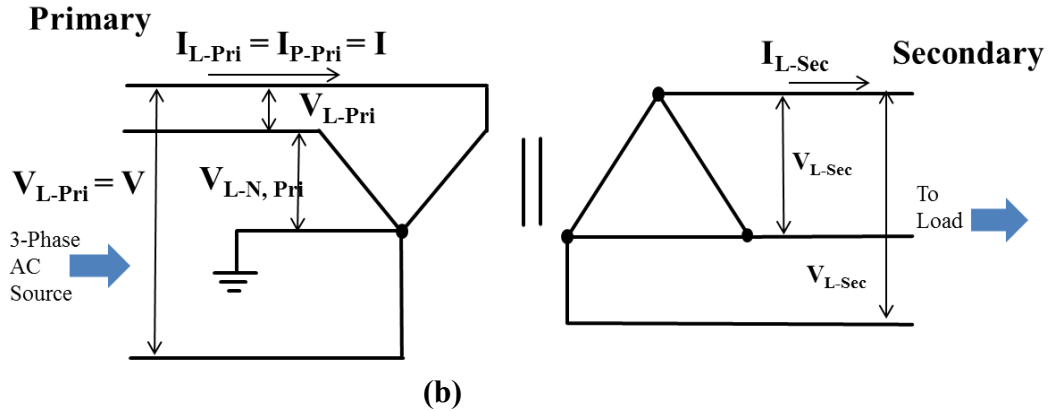


**Figure 1.26:**  $\Delta$ – $Y$ , or Delta – Star, 3-Phase Voltage and Current transformation

### Three Phase Y- $\Delta$ Transformer Configuration

The wiring diagram of a three phase Y- $\Delta$  transformer is shown in Figure 1.27 (a); where the points labeled H0 through H3 represent the “Y” primary terminals and points X1 through X3 represent the  $\Delta$  secondary terminals. The schematic version of this Y- $\Delta$  transformer is represented by Figure 1.27 (b).





**Figure 1.27:** (a) Y-Δ, or Wye - Delta, transformer wiring diagram, (b) Y-Δ, transformer schematic.

As illustrated in Figure 1.27 (b), in a Y-Δ three phase transformer, the phase and line voltages; and phase and line currents can be calculated using the following equations:

$$V_{P-Pri} = V_{L-N, Pri} = \frac{V_{L-Pri}}{\sqrt{3}} = \frac{V}{\sqrt{3}} \quad \text{Eq. 1.34}$$

$$V_{L-Sec} = V_{P-Sec} \quad \text{Eq. 1.35}$$

$$I_{P-Pri} = I_{L-Pri} \quad \text{Eq. 1.36}$$

$$I_{L-Sec} = \sqrt{3} \cdot I_{P-Sec} \quad \text{Eq. 1.37}$$

As illustrated in Figure 1.28, key mathematical formulae for transformations between the primary and secondary voltages and primary and secondary currents are follows:

$$V_{\text{Line - Delta}} = \frac{V_{\text{Line - Y}}}{a\sqrt{3}}$$

OR, 
$$V_{\text{Line - Delta}} = \frac{V}{a\sqrt{3}} \quad \text{Eq. 1.38}$$

$$I_{\text{Line - Delta}} = a\sqrt{3}.I_{\text{Line - Y}}$$

or,  $I_{\text{Line - Delta}} = a\sqrt{3}.I$

Eq. 1.39

Where,

$V_{\text{P-Pri}}$  = Primary phase voltage

$V_{\text{L-Pri}}$  = Primary line voltage =  $V = V_{\text{Line-Y}}$

$V_{\text{P-Sec}}$  = Secondary,  $\Delta$ , phase voltage

$V_{\text{L-Sec}}$  = Secondary,  $\Delta$ , line voltage

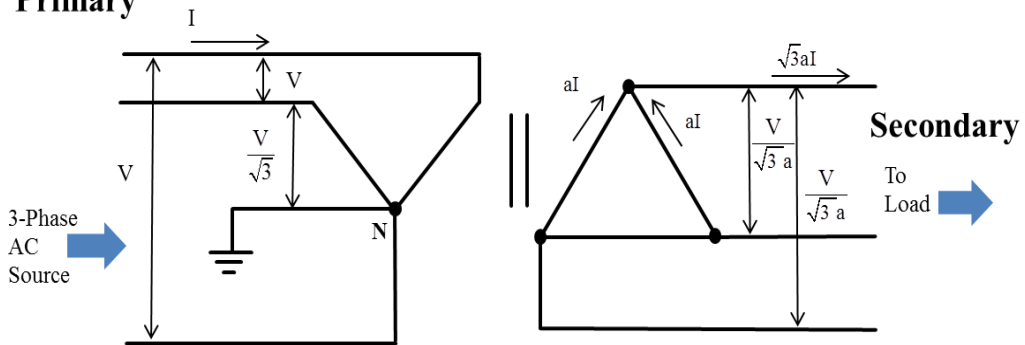
$I_{\text{L-Pri}}$  = Primary line current =  $I = I_{\text{Line-Y}}$

$I_{\text{P-Pri}}$  = Primary phase current

$I_{\text{P-Sec}}$  = Secondary,  $\Delta$ , phase current

$I_{\text{L-Sec}}$  = Secondary,  $\Delta$ , line current

**Primary**

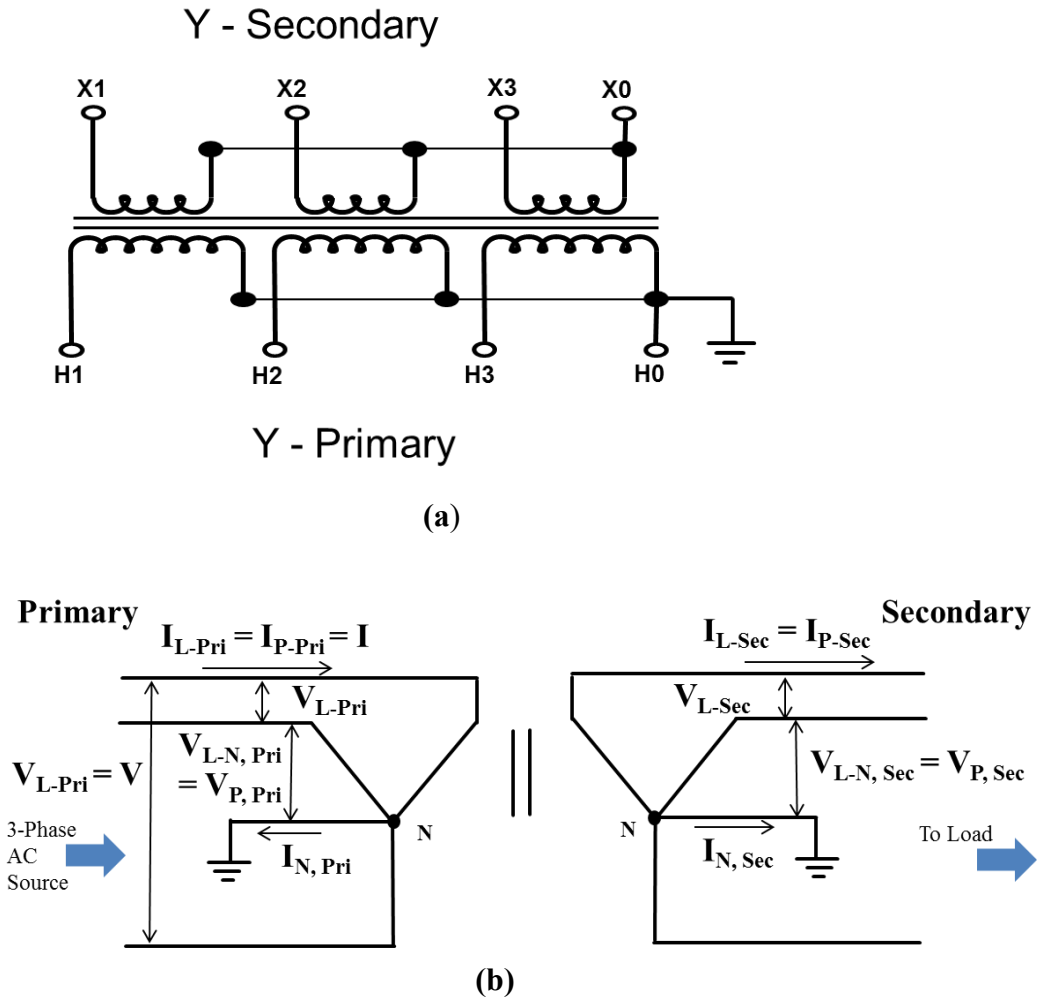


**Figure 1.28:** Y- $\Delta$ , or Star - Delta, 3-Phase Voltage and current transformation schematic

**Three Phase Y-Y Transformer Configuration**

The wiring diagram of three phase Y- Y transformer is shown in Figure 1.29 (a); where the points labeled H0 through H3 represent the “Y” primary terminals and points X0 through X3 represent the Y secondary terminals. Note that the neutrals on primary and secondary windings of Y-Y transformers may or may not be grounded. As shown in Figures 1.29 (a) and (b), the Y-Y transformer, in this specific example, has grounded neutrals on the primary **and** secondary sides. The decision to ground the neutrals, or to leave them “floating,” is premised on specific concerns associated with noise, harmonics and load balancing. Schematic version of this Y-Y transformer is depicted in

Figure 1.29 (b). The term “*floating*,” in the electrical domain implies isolation of a circuit or point from a verified low resistance ground. Floating electrical circuits could belong to an AC or DC system; regardless, such points would exhibit voltages above ground, i.e. +/- 5 VDC, +/- 10 VDC, +/- 24 VDC, 14 VAC, 24 VAC, 110 VAC, etc.



**Figure 1.29:** (a) Y-Y, or Star - Star, 3-Phase transformer wiring diagram (b) Y-Y, or Star-Star, 3-Phase transformer schematic

For a typical Y-Y three phase transformer, as depicted in Figure 1.29 (b), mathematical relationships between the phase and line voltages and phase and line currents are as follows:

$$V_{P-Pri} = V_{L-N, Pri} = \frac{V_{L-Pri}}{\sqrt{3}} = \frac{V}{\sqrt{3}} \quad \text{Gs 03062}$$

$$V_{L-Sec} = \sqrt{3}V_{P-Sec} = \sqrt{3}V_{L-N, Sec} \quad \text{Gs 03063}$$

$$I_{P-Pri} = I_{L-Pri} = I \quad \text{Gs 03064}$$

$$I_{L-Sec} = I_{P-Sec} \quad \text{Gs 03065}$$

\*\*\*\*\*Cu'knwutcvf 'kp'Hi wtg'3052.'hg{'o cyj go c'wecnhqto wcu'hqt"  
 wcpuhqto c'kqpu'dgy ggp'vj g'r tko ct{'cpf "ugeqpf ct{'xqnci gu'cpf 'r tko ct{'cpf "  
 ugeqpf ct{'ewtgpw'ctg'hqmqy u<

$$V_{L-sec} = \frac{V_{L-Pri}}{a} = \frac{V}{a} \quad \text{Gs 03066}$$

$$I_{L-sec} = a.I_{L-Pri} = a.I \quad \text{Gs 03067}$$

Y j gtg."

**X**? "**X<sub>N/Rik</sub>**? "Rtko ct{'hpg'xqnci g'"

**X<sub>N6P.Rik</sub>**? "Rtko ct{'hpg'v'pgwtcn'xqnci g'? "Rtko ct{'r j cug'xqnci g"

**X<sub>N/P.Uge</sub>**? "**X<sub>R/Uge</sub>**? "Ugeqpf ct{'r j cug'xqnci g.'qt.'hpg'v'pgwtcn'ugeqpf ct{' "  
 r j cug'xqnci g"

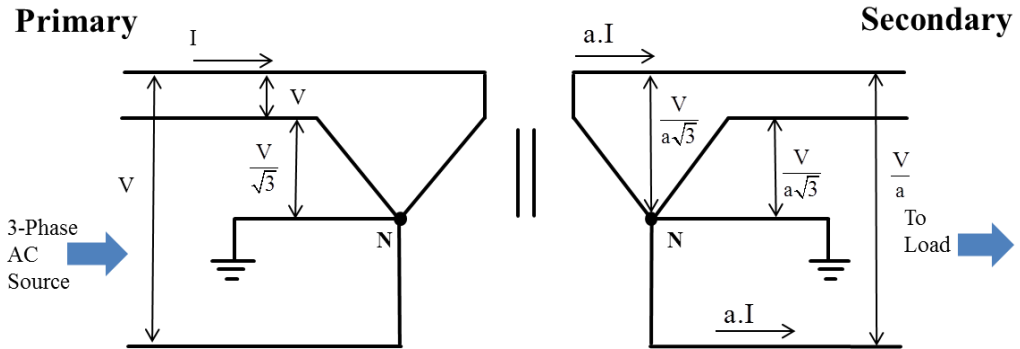
**X<sub>N/Uge</sub>**? "Ugeqpf ct{'hpg'xqnci g"

**K**? "Rtko ct{'hpg'ewtgpv'? "**K<sub>N/Rik</sub>**"

**K<sub>N/Rik</sub>**? "Rtko ct{'r j cug'ewtgpv'"

**K<sub>N/Uge</sub>**? "Ugeqpf ct{'r j cug'ewtgpv'"

**K<sub>N/Uge</sub>**? "Ugeqpf ct{'hpg'ewtgpv'"

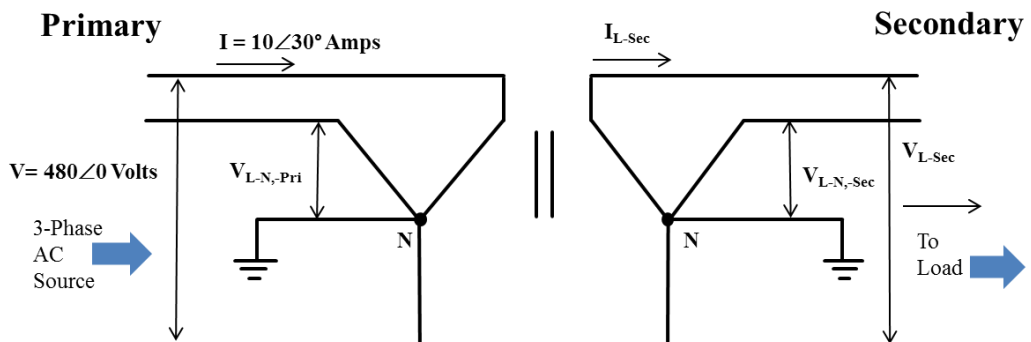


**Figure 1.30:** Y –Y, or Star–Star, voltage and current transformation

**Example 1.7**

Consider the power distribution system shown in the schematic below. Determine the following unknown parameters on the, **Y**, load side of the transformer given that the turns ratio is 2:1:

- a)  $|I_{L-Sec}|$  = Magnitude of load or secondary line current
- b)  $|I_{P-Sec}|$  = Magnitude of secondary phase current or load phase current
- c)  $|V_{P-Pri}|$  = Magnitude of phase voltage on the source or primary side of the transformer
- d)  $|V_{L-Sec}|$  = Magnitude of line voltage on the load or secondary side of the transformer
- e)  $|V_{P-Sec}|$  = Magnitude of phase voltage on the load or secondary side of the transformer
- f)  $|V_{L-N, Sec}|$  = Magnitude of line to neutral voltage on the load or secondary side of the transformer





**Solution:**

a) Using Eq. 1.45

$$I_{P-Pri} = I_{L-Pri} = 10 \angle 30^\circ \text{ Amps}$$

$$|I_{L-sec}| = a \cdot |I_{L-Pri}| = a \cdot |I| = (2) \cdot |10 \angle 30^\circ| = (2) \cdot 10 \text{ Amps} = 20 \text{ Amps}$$

b) According to Eq. 1.43 and as computed in part (a):

$$I_{P-Sec} = I_{L-Sec} = 20 \angle 30^\circ \text{ Amp}$$

$$\therefore |I_{P-Sec}| = 20 \text{ Amps}$$

c) According to Eq. 1.40:

$$|V_{P-Pri}| = |V_{L-N, Pri}| = \frac{|V_{L-Pri}|}{\sqrt{3}} = \frac{|V|}{\sqrt{3}} = \frac{|480 \angle 0^\circ|}{\sqrt{3}} = 277 \text{ V}$$

d) According to Eq. 1.41:

$$|V_{L-sec}| = \frac{|V_{L-Pri}|}{a} = \frac{|V|}{a} = \frac{|480 \angle 0^\circ|}{2} = \frac{480}{2} = 240 \text{ Volts}$$

e) As computed in part (d),  $|V_{L-Sec}| = 240 \text{ V}$ . Then, using Eq. 1.41:

$$V_{L-Sec} = \sqrt{3} V_{P-Sec} = \sqrt{3} V_{L-N, Sec}$$

And, by rearranging Eq. 1.41

$$V_{P-Sec} = V_{L-N, Sec} = \frac{V_{L-Sec}}{\sqrt{3}}$$

$$\therefore |V_{P-Sec}| = |V_{L-N, Sec}| = \frac{|V_{L-Sec}|}{\sqrt{3}}$$

$$\text{and, } |V_{P-Sec}| = \frac{240}{\sqrt{3}} = 139 \text{ V}$$

f) According to the rearranged version of Eq. 1.27, in part (e):

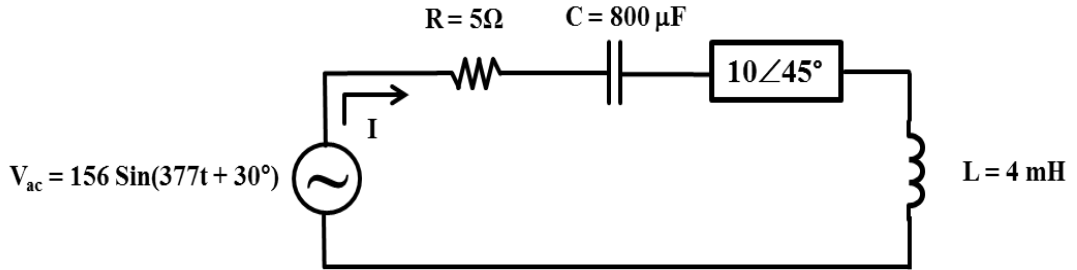
$$V_{P-Sec} = V_{L-N, Sec} \text{ and,}$$

$$\text{and, } |V_{P-Sec}| = |V_{L-N, Sec}| = \frac{240}{\sqrt{3}} = 139 \text{ V}$$

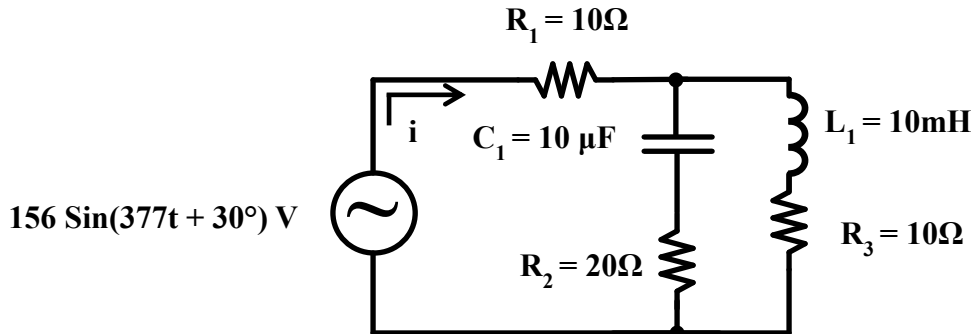
### Self-Assessment Problems & Questions – Segment 1

1. A plating tank with an effective resistance of 100 ohm is connected to the output of a full-wave rectifier. The AC supply voltage is  $340V_{\text{peak}}$ . Determine the amount of time, in hours, it would take to perform 0.075 faradays worth of electroplating?

2. Determine the source current  $I_{\text{rms}}$  in the AC circuit below.

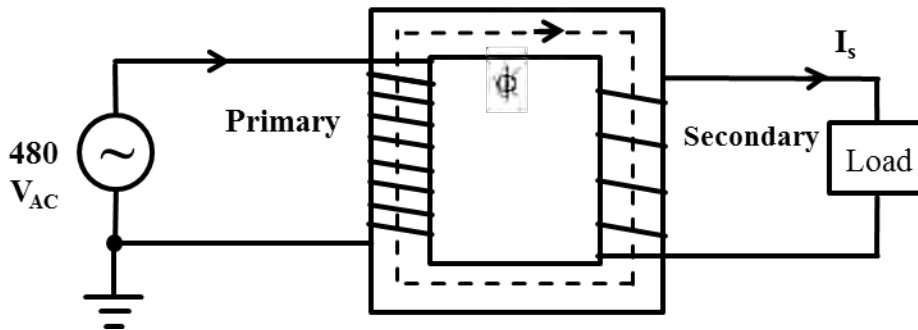


3. Calculate the impedance  $Z_{\text{EQ}}$  as seen by the AC voltage source in the circuit below:

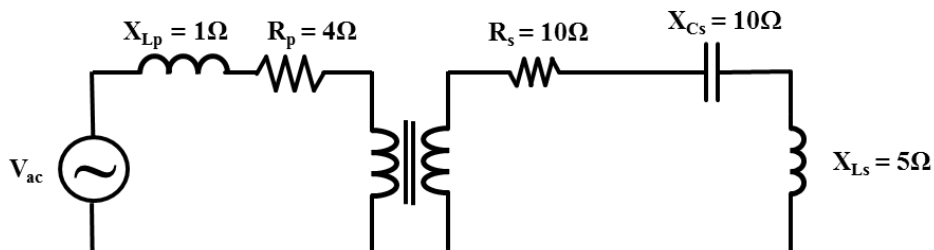


4. A single phase 1 kVA resistive load, designed to operate at 240 VAC, has to be powered by a 480 VAC source. A transformer is applied as shown in the diagram below. Answer the following questions associated with this scenario:

- Would the transformer be connected in a “step up” configuration or a “step down” configuration?
- When installing the transformer, what turns ratio,  $a$ , should it be connected for?
- What would be the secondary current,  $I_s$ , when the load is operating at full capacity?
- What would be the primary current at full load?



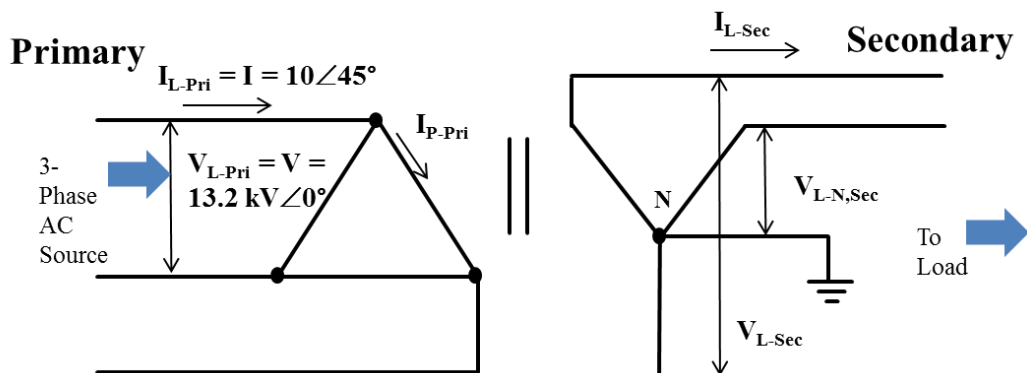
5. Calculate the equivalent impedance as seen from the vantage point of the AC source  $V_{ac}$  in the circuit shown below. The transformer in the circuit is assumed to be ideal. The values of the primary and secondary circuit elements are:  $X_{Lp} = 1\Omega$ ,  $R_p = 4\Omega$ ,  $R_s = 10\Omega$ ,  $X_{Ls} = 5\Omega$ ,  $X_{Cs} = 10\Omega$ ,  $N_p = 100$ ,  $N_s = 200$ .



6. The no load voltage at the main switch yard of a manufacturing facility is 13,400 V<sub>AC</sub>. The voltage regulation of the main switch yard is 4%. What is the rated full load voltage that is most likely to be measured on the load side of the main switch yard?

7. Consider the power distribution system shown in the schematic below. Determine the following unknown parameters on the, **Y**, load side of the transformer given that the turns ratio is 2:1:

- a)  $|I_{L-Sec}|$  = Magnitude of load or secondary line current
- b)  $|I_{P-Sec}|$  = Magnitude of secondary phase current or load phase current
- c)  $|V_{P-Pri}|$  = Magnitude of Phase voltage on the source or primary side of the transformer
- d)  $|V_{L-Sec}|$  = Magnitude of Line voltage on the load or secondary side of the transformer
- e)  $|V_{P-Sec}|$  = Magnitude of Phase voltage on the load or secondary side of the transformer
- f)  $|V_{L-N, Sec}|$  = Magnitude of Line to neutral voltage on the load or secondary side of the transformer



## **Segment 2**

### **AC vs. DC Power**

#### **Introduction**

This segment begins with an introduction to concepts of energy, DC power, and DC system efficiency. DC power and efficiency discussion sets the stage for the more complex concepts of single phase AC power and AC energy. And, discussion of single phase AC serves as a suitable segue for consideration of the more complex subject of three phase AC power. Significance of efficiency and energy will be explained within the context DC power and AC power. Once the basic understanding of AC and DC power is clear, we will be better poised to examine the flow of power and energy from the point of generation to the point of consumption. The flow of power and energy from some common sources such as steam and water to electricity are described, with illustrations in form of example problems and self-assessment problems. The flow of power from utility to pumps is discussed in the “Wire to Water” section.

#### **Power**

Power can be defined in multiple ways. Power can be defined as the rate of performance of work, rate of generation of energy, and even as rate of application of torque. This is congruent with the fact that work, energy and torque are equivalent. All three can be quantified or measured - in the Metric or SI system - as N-m (Newton- meters) or J (Joules). In the US unit realm, work, energy and torque can be measured in ft-lbf, in-lbf, BTU (British Thermal Unit), Hp-hr (Horsepower-hour), etc. Since power is a derivative of energy with respect to time, its units in the SI or Metric realm are N-m/s (Newton- meters/second) or J/s (Joules/second). And, the units for power in the US domain are ft-lbf/s (foot-pound force/second), in-lbf/s (in-pound force/second), BTU/s (British Thermal Units/second) and hp (horsepower).

The power and energy relationships stated above gain practical significance when stated mathematically as follows:

$$E = W = \tau \qquad \text{Eq. 2.1}$$

Where, **E** represents energy, **W** represents work and  $\tau$  denotes torque.

Two common forms of energy in the mechanical and civil realm are as follows:

$$\text{Potential Energy} = mgh \quad \text{Eq. 2.2}$$

$$\text{Kinetic Energy} = \frac{1}{2}mv^2 \quad \text{Eq. 2.3}$$

$$\text{Work} = F \times d \quad \text{Eq. 2.4}$$

$$\tau = F \times l \quad \text{Eq. 2.5}$$

Where,

**m** = mass of an object, in kg (SI units system)

**g** = Acceleration due to gravity =  $9.8\text{m/s}^2$  (SI units system)

**h** = elevation of the object, in meters

**v** = Velocity in m/s

**F** = Force in N (Newtons)

**d** = Distance, in meters, over which the object is pushed or pulled by force, F

**l** = Moment arm, or perpendicular distance, in meters, from the fulcrum, at which the force is applied

In order to illustrate that potential energy, kinetic energy, work and torque can ultimately be measured in joules and kWh (unit for electrical energy), let's expand Equations 4.2 through 4.5 into N-m and Joules:

$$\text{Potential Energy} = mgh = (\text{kg})\left(\frac{\text{m}}{\text{s}^2}\right)(\text{m}) = \text{N}\cdot\text{m} = \text{J}$$

$$\begin{aligned} \text{Kinetic Energy} &= \frac{1}{2}mv^2 = \frac{1}{2}(\text{kg})\left(\frac{\text{m}}{\text{s}}\right)^2 = \frac{1}{2}(\text{kg})\left(\frac{\text{m}}{\text{s}^2}\right)(\text{m}) \\ &= \frac{1}{2}\text{N}\cdot\text{m} = \frac{1}{2}\text{J} \end{aligned}$$

$$\text{Work} = F \times d = \text{N}\cdot\text{m} = \text{J}$$

$$\tau = F \times l = \text{N}\cdot\text{m} = \text{J}$$

Since the four forms of energy correspond and correlate to the basic unit of energy, Joule, the basic unit of electrical power can be derived as:

$$\text{Power} = \frac{\text{Energy}}{\text{Time}} = \left( \frac{\text{J}}{\text{s}} \right) = \text{Watt};$$

and 1,000 Watts = 1kW, or 1 kilo-Watt

Conversely,

$$\text{Energy} = (\text{Power}).(\text{Time}) = (\text{W}).(\text{s}) = \left( \frac{\text{J}}{\text{s}} \right)(\text{s}) = \text{J}$$

Or, 1kWh of Energy = (1000W).(1h)

$$= (1000) \left( \frac{\text{J}}{\text{s}} \right) (3600\text{s}) = 3600 \text{ kJ}$$

Now, in order to demonstrate the interchangeability between electrical energy and hydrocarbon fuel (i.e. fuel oil, natural gas, propane, gasoline, etc.), let's extrapolate the conversion of 1kWh into the Btu's and dekatherms:

Since 1055 J = 1 Btu,

$$\begin{aligned} 1\text{kWh of Energy} &= 3600 \text{ kJ} = 3,600,000 \text{ J} = \left( \frac{3,600,000 \text{ J}}{1055 \text{ J/Btu}} \right) \\ &= 3,412 \text{ Btu} \end{aligned}$$

And since 1,000,000 Btu = 1MMBtu = 1 DT,

$$\begin{aligned} 1\text{kWh of Energy} &= 3600 \text{ kJ} = 3,412 \text{ Btu} = 0.003412 \text{ MMBtu} \\ &= 0.003412 \text{ DT (Dekatherm)} \end{aligned}$$

Following the energy and power unit conversion methods illustrated above, one can perform operating energy cost comparison between alternative pieces of equipment.

### **Example 2.1**

Which of the following two water heaters would cost the least to operate, on annual cost basis, under the given assumptions?

#### **A. Electric Water Heater:**

Estimated annual energy consumption: **5000 kWh**

Efficiency: **100%**

Cost Rate: **\$0.10/kWh**

**B. Natural Gas Water Heater:**

Estimated annual energy consumption: **Same as the Electric water heater**

Efficiency: **100%**

Cost Rate: **\$10.87/DT**

**Solution:**

Since the cost rate for electrical energy and the annual electrical energy consumption for the electric water heater are given:

Total annual cost for operating the *electric water heater*

$$= (5000 \text{ kWh}).(\$0.10/\text{kWh}) = \$500$$

The annual energy consumption by the *gas water heater*, assumed to be the same as the electric water heater = **5000 kWh**

Then, the annual energy consumption by the gas water heater, in DT or MMBtu would be

$$\begin{aligned} &= (5000 \text{ kWh}).(3412\text{Btu/kWh})(1\text{DT}/1,000,000\text{Btu}) \\ &= \mathbf{17.06 \text{ DT}.} \end{aligned}$$

Since the natural gas cost rate is given as **\$10.87/DT**, the annual operating cost for the gas water heater would be = **(17.06 DT).(\$10.87/DT) = \$185.44.**

**Answer:** The gas water heater would cost substantially less to operate than the electric water heater.

**Electric Motor Horsepower Required to Move/Convey Mass<sup>1</sup>**

Conveyance of material through powered conveyors, aside from the selection and design of other pieces of equipment, requires sizing and specification of conveyor motors.

Two simple formulas that can be used to assess the brake horsepower required to move *“loose”* (i.e. aggregate bulk materials like sand, flour, grain, sugar,

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<sup>1</sup>Thermodynamics Made Simple for Energy Engineers, By: S. Bobby Rauf)



etc.) or discrete “*unit*” mass (i.e. solid objects like steel parts, fabricated mechanical components, wooden objects, etc.) are listed below:

$$\mathbf{P} = \mathbf{F} \cdot \Delta \mathbf{v} \quad \text{Eq. 2.6}$$

$$\mathbf{F} = \quad \Delta \mathbf{v} \quad \text{Eq. 2.7}$$

Where,

**P** = Power in J/s or Watts = Brake horsepower required at the motor shaft.

**F** = Linear force required to move the conveyer (or conveyor belt), loaded with the aggregate or unit mass to be moved

**Δv** = Change in the velocity of aggregate or unit mass being conveyed, in the direction of the applied force and in the direction the material is being moved.

= Mass flow rate.

Application of this method for assessing the size of a conveyor motor is illustrated through Example 2.2

### **Example 2.2**

Sand drops at the rate of 20,000 kg/min onto a conveyor belt moving with a velocity of 2.0 m/s.

- (a) What force is required to keep the belt moving?
- (b) What is the minimum motor size that should be specified for this application?

### **Solution:**

$$\begin{aligned} \text{(a) } \mathbf{F} &= \quad \Delta \mathbf{v} = (20,000 \text{ kg/min}) \cdot (2.0 \text{ m/s} - 0 \text{ m/s}) \cdot (1/60 \text{ min/s}) \\ &= 666.67 \text{ kg} \cdot \text{m/s}^2 \text{ or, } \mathbf{F} = 666.67 \text{ N} \end{aligned}$$

$$\begin{aligned} \text{(b) } \mathbf{Power} = \mathbf{P} &= \mathbf{F} \cdot \mathbf{v} = (666.67 \text{ N}) \cdot (2 \text{ m/s}) \\ &= 1333.3 \text{ N} \cdot \text{m/s} = 1333.3 \text{ J/s} = 1333.3 \text{ Watts} \end{aligned}$$

Since 746 W = 1 hp, minimum motor horsepower required:

$$= 1333.3 / 746 = 1.79 \text{ hp}$$

∴ Specify a 2 hp (standard size) motor

## **DC Power**

Power in direct current realm is equivalent to power in the mechanical realm, for most practical purposes. DC electrical power is also referred to as the “real” power. DC power is called real because, unlike AC power, it can be transformed entirely into work or other forms of energy, i.e. heat energy, potential energy, kinetic energy, etc. Such transformation can be assessed through power and energy relationships discussed earlier in this segment. Eq. 2.8 below represents one way to correlate DC, real, power to mechanical work performed.

$$W = \text{Energy} = P.t \qquad \text{Eq. 2.8}$$

Where,

**W** = Mechanical work performed under DC electrical power, measured in Joules (or N-m).

**P** = DC electrical power, in Watts (or J/s).

**t** = DC power application duration, in seconds (s)

DC power can be defined mathematically as stipulated in Eq. 2.9.

$$P = V.I \qquad \text{Eq. 2.9}$$

Where,

**P** = DC electrical power, in Watts (or J/s)

**V** = DC voltage measured in volts (V)

**I** = DC current measured in amps (A)

We can quantify mechanical work in terms of voltage, current and time by substituting Eq. 2.9 into Eq. 2.8:

$$W = \text{Energy} = V.I.t \qquad \text{Eq. 2.10}$$

In the SI or Metric unit system, DC power or “real” power is traditionally measured in Watts, kW, MW, GW, TW ( $10^{12}$  W). Where k = 1000, M = 1000,000, G = 1 billion and T = 1 trillion.

Some of the more common power conversion factors that are used to convert between SI System and US system of units are listed below:

$$1.055 \text{ kJ/s} = 1.055 \text{ kW} = 1 \text{ BTU/s}$$

$$\text{One hp} = 746 \text{ Watts} = 746 \text{ J/s} = 746 \text{ N-m/s} = 0.746 \text{ kW} = 550 \text{ ft-lbf/sec}$$

In the SI or Metric unit system, DC energy or “real” energy is traditionally measured in Wh, kWh, MWh, GWh, TWh ( $10^{12}$  Wh).

Some mainstream conversion factors that can be used to convert electrical energy units within the SI realm or between the SI and US realms are referenced below:

$$1000 \text{ kW} \times 1\text{h} = 1 \text{ MWh}$$

$$1 \text{ BTU} = 1055 \text{ J} = 1.055 \text{ kJ}$$

$$1 \text{ BTU} = 778 \text{ ft-lbf}$$

$$1 \text{ hp} \times 1\text{hour} = 1 \text{ hp-hour}$$

### **Example 2.3**

An automobile is parked with parking lights left on for one hour. The automobile battery is rated  $12V_{DC}$  and the lamps are incandescent. If all of the parking lamps are drawing a combined current of 4A, what is the total energy consumed by the parking lights, in the form of heat and light?

#### **Solution:**

Apply Eq. 2.10:

$$\begin{aligned} \text{Energy} &= V \cdot I \cdot t = (12V) \cdot (4A) \cdot (1\text{hr}) = (48W) \cdot (3600 \text{ s}) \\ &= \left( 48 \frac{\text{J}}{\text{s}} \right) (3600\text{s}) = 172,800 \text{ J} \end{aligned}$$

### **Single Phase AC Power**

AC, single and three phase, was introduced in Segment 1. In Segment 1, we also discussed the differences between DC, single phase AC and three phase AC. In this segment, we will focus mostly on single phase and three phase power. While single phase AC power is primarily power or rate of performance of work, it differs from DC power, real power or mechanical power in the fact that it is not devoted *entirely* to performance of work or conversion into other forms of energy. In most cases, a percentage of AC

power is “sequestered” in the form of electromagnetic energy and power. This percentage of AC power is not available for performance of mechanical work or conversion to other forms of energy; instead, it is dedicated toward charging and discharging inductances and capacitances in the AC electrical system. Yet, the utility or the power company must generate and supply the *entire* AC power to sustain *all* customers regardless of the size of inductances and capacitances in their systems.

The term “single phase AC” implies that the AC power source consists of one energized or higher electrical potential conductor or terminal. The other conductor, wire or terminal serves as a neutral and is typically connected to building or power distribution system ground. Mechanical engineers, with thermodynamic background, could view the energized line or terminal as the “heat source” and could apply the analogy of “heat sink” to the neutral or ground. This analogy is premised on the fact that, for heat engines to perform work through steam turbines, the superheated steam must have a heat source and a heat sink to traverse between.

As introduced briefly in Segment 1, total AC power is called Apparent Power and is denoted by **S**. Note that the apparent power symbol **S** is bold faced. The bold font signifies the fact that apparent power is a *vector* and can be “completely” represented by a *complex number* that consists of a *magnitude* and an *angle*.

Fundamental mathematical definition of single phase AC power is represented in terms of Eq. 2.11 and Eq. 2.12. The magnitude and angle of single phase AC apparent power can be determined through equations 2.13 and 2.14, respectively. The vector values of single phase AC voltage and current can be calculated using Eq. 2.15 and Eq. 2.16, respectively. The magnitudes of single phase AC voltage and current can be calculated using Eq. 2.17 and Eq. 2.18, respectively. Values of P, Q and |S| can also be calculated in terms of  $V_{RMS}$ , **R**, **X** and **Z** as shown in Eq. 2.19, Eq. 2.20 and Eq. 2.21.

$$\vec{S} = \vec{V} \cdot \vec{I}^* \quad \text{Eq. 2.11}$$

$$\vec{S} = P + jQ \quad \text{Eq. 2.12}$$

$$|\vec{S}| = |\vec{V}| \cdot |\vec{I}| \quad \text{Eq. 2.13}$$

$$\text{Angle of S} = \angle\theta_s = \text{Tan}^{-1}\left(\frac{Q}{P}\right) \quad \text{Eq. 2.14}$$

$$\vec{V} = \frac{\vec{S}}{\vec{I}^*} \quad \text{Eq. 2.15}$$

$$\vec{I}^* = \frac{\vec{S}}{\vec{V}} \quad \text{Eq. 2.16}$$

$$|V| = \frac{|S|}{|I|} \quad \text{Eq. 2.17}$$

$$|I| = \frac{|S|}{|V|} \quad \text{Eq. 2.18}$$

$$P = \frac{V_{RMS}^2}{R} \quad \text{Eq. 2.19}$$

$$Q = \frac{V_{RMS}^2}{X} \quad \text{Eq. 2.20}$$

$$|S| = \frac{|V_{RMS}|^2}{|Z|} \quad \text{Eq. 2.21}$$

Where,

$\vec{S}$  = Complex, vector, representation of single phase AC power,

measured in VA, or Volt-Amperes, kVA or MVA

$\vec{V}$  = Complex, vector, representation of single phase AC voltage.

measured in volts

$\vec{I}$  = Complex, vector, representation of single phase AC current.

measured in amperes

$|V|$  = Magnitude or absolute value of single phase AC voltage, in volts

$|V_{RMS}|$  = Magnitude of the RMS value of single phase AC voltage

$|I|$  = Magnitude or absolute value of single phase AC current, in amps

$|S|$  = Magnitude or absolute value of single phase AC apparent power, measured in VA, or Volt-Amperes, kVA or MVA

$\theta$  = Angle of apparent power **S**.

**P** = Real power, or real component of apparent power **S**, measured in Watts, kW or MW.

**Q** = Reactive Power, or imaginary component of apparent power **S**, measured in VARs, or, Volt-Amperes Reactive, kVAR or MVAR.

**j** = Unit vector, with a magnitude of “1” or unity, portending an angle of 90° with respect to the x-axis. As explained in Segment 1, unit vector **j** can be written in polar or phasor form as  $1\angle 90^\circ$ .

**X** = Reactance, in  $\Omega$

**R** = Resistance in  $\Omega$

$|\bar{Z}|$  = Magnitude or absolute value of AC impedance in  $\Omega$

Since apparent power **S** is represented as a vector with horizontal component “**P**” and vertical component “**Q**,” in Eq. 2.12, an alternate method for calculating the magnitude of apparent power **S** (vector) can be derived from the Pythagorean Theorem as stipulated by Eq. 2.22. The angle of apparent power vector **S** could still be computed using Eq. 2.14.

$$|\bar{S}| = \text{Magnitude of AC Apparent Power} = \sqrt{P^2 + Q^2} \quad \text{Eq. 2.22}$$

**Power Factor:** A concept that is inherently important in most analysis and consideration of AC power is “Power Factor.” While power factor is

discussed, in detail, in Segment 3, as a prelude to that detailed discussion and to complete the discussion of single phase AC power, mathematical definition of power factor is stated below in form of equations 2.23 and 2.24.

$$\text{PF} = \text{Power Factor} = \frac{|P|}{|S|} \quad \text{Eq. 2.23}$$

$$\text{PF} = \text{Power Factor} = \text{Cos}(\theta_V - \theta_I) \quad \text{Eq. 2.24}$$

Where,

$|P|$  = Magnitude of the real component of AC apparent power

$|S|$  = Magnitude of the overall AC apparent power

$\theta_V$  = Angle of AC voltage

$\theta_I$  = Angle of AC current

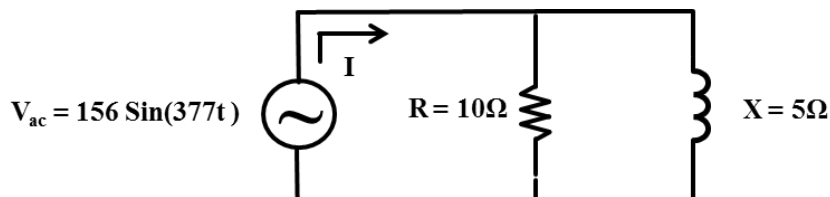
### Example 2.4

A  $156\text{Sin}377t$  sinusoidal voltage is connected across a load consisting of a parallel combination of a  $10\ \Omega$  resistor and a  $5\ \Omega$  inductive reactance.

- (a) Determine the real power dissipated by the resistor.
- (b) Determine the reactive power sequestered in a  $5\ \Omega$  parallel reactance.
- (c) Calculate the total apparent power delivered to this parallel R and X circuit by the AC voltage source.

### Solution:

The circuit diagram for this scenario would be as depicted below:



- a) We can apply Eq. 2.19 to determine the power dissipated or consumed in the  $10\ \Omega$  resistor. However, we must first derive the  $V_{\text{RMS}}$  from the given AC voltage of  $156\text{Sin}377t$ . This is due to the fact that, by convention,

coefficient 156 stated in the give AC voltage function of  $156\sin 400t$  is the peak or maximum voltage,  $V_m$ .

As discussed in Segment 1 and stipulated by Eq. 1.3:

$$V_m = \sqrt{2}V_{RMS} \quad \text{or} \quad V_{RMS} = \frac{V_m}{\sqrt{2}}$$

$$\therefore V_{RMS} = \frac{156}{\sqrt{2}} = 110.3V$$

Then, according to Eq. 2.19:

$$P = \frac{V_{RMS}^2}{R} = \frac{(110.3)^2}{10} = 1217 \text{ Watts}$$

- b) Apply Eq. 2.20 to determine the reactive power sequestered in the  $5\Omega$  parallel reactance.

$$Q = \frac{V_{RMS}^2}{X} = \frac{(110.3)^2}{5} = \frac{12,166}{5} = 2433 \text{ VAR}$$

- c) Apply Eq. 2.12 to calculate the total apparent power  $S$  delivered to this parallel R and X circuit by the AC voltage source.

$$\vec{S} = P + jQ = 1217 + j2433 = 2720 \angle 63.4^\circ W$$

**Note:** We can also apply Eq. 2.22 to verify the magnitude of the total apparent power  $S$  delivered to this parallel R and X circuit:

$$\begin{aligned} |S| &= \text{Magnitude of AC Apparent Power} = \sqrt{P^2 + Q^2} \\ &= \sqrt{1217^2 + 2433^2} = 2720 \text{ VA} \end{aligned}$$

Note that  $jQ$  reactive power entity is entered into the apparent power calculation as  $+jQ$  because of the fact that inductance in the given AC circuit results in positive impedance contribution or “ $+jX$ .”



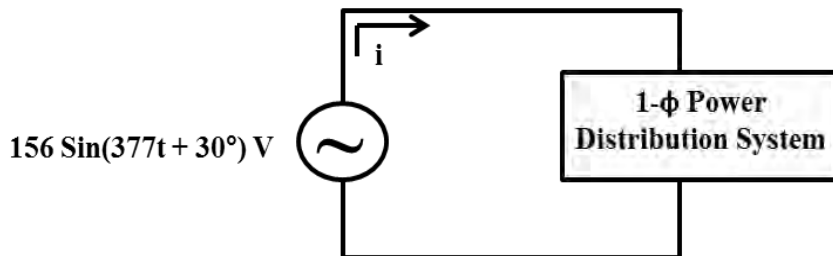
**Ancillary:** Reader is encouraged to verify the apparent power of 2720 VA by applying equation 2.21. **Hint:** The  $Z$ , in this case must be computed through parallel combination of  $R$  and  $Z_L$  as shown below:

$$|Z| = \left| \frac{R \cdot Z_L}{R + Z_L} \right| \qquad \text{Eq. 2.25}$$

**Example 2.5**

The AC circuit shown below depicts a simplified, single phase, one-line diagram of a “special purpose” power generating station. Assume that there is no voltage drop between the generator and the power distribution system. The line current is measured to be 1,000 amps, rms. Calculate the following if the power factor is known to be 0.9:

- a) Magnitude of the apparent power presented to the power distribution system.
- b) Magnitude of the real power presented to the power distribution system.



**Solution:**

a) Magnitude of the apparent power presented to the power distribution system:

$$|S|_{1-\phi} = \sqrt{(P)^2 + (Q)^2} = |V \cdot I^*|$$

Since we are interested in the magnitude of the single phase AC power:

$$|S|_{1-\phi} = |V \cdot I^*| = |V| \cdot |I|$$

Where,  $V$  and  $I$  are RMS values. Note that, by convention, the 156V in the given AC voltage function is  $V_p$ , or the peak value.

$$V_{\text{rms}} = \frac{V_p}{\sqrt{2}} = \frac{156}{\sqrt{2}} = 110 \text{ V}$$

$$\begin{aligned} |S|_{1-\phi} &= |V \cdot I^*| = |V| \cdot |I| \\ &= (110 \text{ V}) \cdot (1,000 \text{ amps}) = 110,000 \text{ VA} \\ &= 110 \text{ kVA} \end{aligned}$$

b) Magnitude of the real power presented to the power distribution system can be determined by rearranging and applying equation 2.23 as follows:

$$\text{PF} = \text{Power Factor} = \frac{|P|}{|S|} \quad \text{Eq. 2.23}$$

$$|P| = |S| \cdot \text{PF} = (110 \text{ kVA}) \cdot (0.9) = 99.34 \text{ kW}$$

### **Three Phase AC Power**

As explained in Segment 1, while single phase AC offered substantial advantages over DC in terms of transmission of power from power generating stations to consumers, three phase AC took electrical power transmission to an even higher level of efficiency and effectiveness. Having explored the concepts, principles analytical techniques, equations and conventions associated with single phase AC, in the last section, we are now adequately prepared to explore basic yet practical concepts, principles analytical techniques, equations and conventions associated with three phase AC. Because conveyance and transformation of electrical energy into mechanical and other forms of energy is more effective and efficient in three phase configuration than single phase configuration, larger loads and large transfers of electrical power and energy are mostly conducted in three phase formation. Therefore, we find most large motors and other types of loads, with a few exceptions, in industrial and commercial facilities, to be three phase. Analytical techniques, mathematical formulas, schematics and other types of three phase drawings tend to be more complex than their single phase counterparts. Due to the profundity and complexity of three phase AC, we will limit our discussion of three phase AC power in this section to the basic level.

Efficient and effective application of three phase AC requires that the load on each of the three phases be as equal as possible. When the loads on each of the three phases are equal, three phase AC system is said to be balanced.

Basic three phase power formulas are listed below:

$$\bar{S}_{3-f} = \text{Three phase apparent power} = \sqrt{3}\bar{V}_{L-L}\bar{I}_L^* \quad \text{Eq. 2.27}$$

$$|S_{3-\phi}| = \sqrt{3}|V_{L-L}||I_L^*| \quad \text{Eq. 2.28}$$

$$\bar{S}_{3-f} = 3. \bar{S}_{1-f} \quad \text{Eq. 2.29}$$

$$|\bar{S}_{3-f}| = 3|\bar{S}_{1-f}| = 3\left(\frac{V_{L-L} \cdot I_L}{\sqrt{3}}\right) = \sqrt{3}V_{L-L} \cdot I_L, \text{ for three}$$

phase Y and D circuits.

$$\text{Eq. 2.30}$$

Where,

$\bar{S}_{3-f}$  = Complex, vector, representation of three phase AC power,

measured in VA, or Volt-Amperes, kVA or MVA.

$|\bar{S}_{3-f}|$  = Magnitude of three phase AC power, measured in VA,

or Volt-Amperes, kVA or MVA.

$\bar{S}_{1-f}$  = Complex, vector, representation of one of the three phases

of AC power, measured in VA, or Volt-Amperes, kVA or MVA.

$|\bar{S}_{1-f}|$  = Magnitude of one of the three phases of AC power, measured

in VA, or Volt-Amperes, kVA or MVA.

$\bar{V}_{L-L}$  = Complex, vector, representation of line to line, or phase to

phase, RMS voltage, measured in volts.

$|\bar{V}_{L-L}|$  = Magnitude of the Line to Line, or Phase to Phase, RMS

voltage, measured in volts.

$\bar{I}_L$  = Complex, vector, representation of line RMS current,

with magnitude measured in amperes, typically, using a

clamp-on ammeter or a CT, Current Transformer.

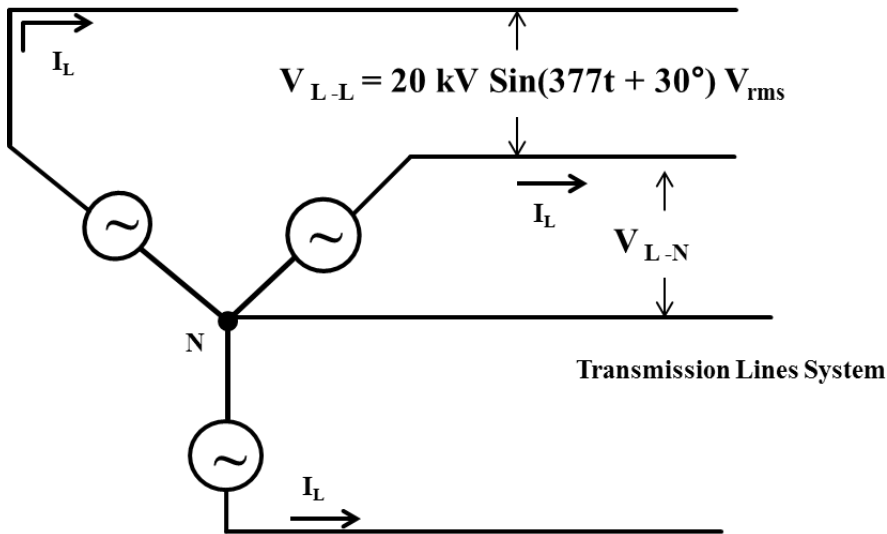
$\vec{I}_L^*$  = Conjugate of vector, representation of line RMS current. Sign of the angle of the conjugate current is opposite of the sign of the original current.

$|\vec{I}_L^*| = |\vec{I}_L|$  = Magnitude of the RMS value of the line current.

**Example 2.6**

The AC circuit shown below depicts a three phase, one-line schematic of a hydroelectric power generating station, modeled after the Three Gorges Dam, China. Assume that there is no voltage drop between the generator and the primary side of the transmission system transformer. The line current is measured to be **22,453 amps**, RMS. Calculate the following if the power factor is known to be **0.9**:

- a) Magnitude of the apparent power presented to the transmission lines.
- b) Magnitude of the real power presented to the transmission lines.



**Solution:**

- (a) Magnitude of the apparent power presented to the transmission lines:  
Note that the AC voltage function is specified in RMS form and not in “peak” or “maximum” form. Therefore, we don’t need to derive the RMS voltage. The line current is given in RMS form, as well.

According to Eq. 2.30:

$$|\bar{S}_{3-f}| = 3|\bar{S}_{1-f}| = 3\left(\frac{V_{L-L} \cdot I_L}{\sqrt{3}}\right) = \sqrt{3}V_{L-L} \cdot I_L, \text{ for three}$$

phase Y and D circuits.

Therefore,

$$\begin{aligned} |\bar{S}_{3-f}| &= \sqrt{3}V_{L-L} \cdot I_L = \sqrt{3}(20,000V) \cdot (22,450A) = 777,690,813VA \\ &= 777,690 \text{ kVA} = 777.69 \text{ MVA} \end{aligned}$$

(b) Magnitude of the real power presented to the transmission lines can be determined by rearranging and using Eq. 2.23:

$$\text{PF} = \text{Power Factor} = \frac{|P|}{|S|}$$

$$\text{or, } |P| = \text{PF} \cdot |S| = (0.9) \cdot (777.69\text{MVA}) = 700\text{MW}$$

## Efficiency

Efficiency is defined, generally, as the ratio of output to input. The output and input, in general, could be in form of power, energy, torque or work. Efficiency assumes a more specific definition when considered in the context of a specific form of energy, work or power. The concept of efficiency, when applied in the electrical engineering domain, typically, involves power or energy.

In electrical engineering, when **power** is the subject of analysis, efficiency is defined as follows:

$$\text{Efficiency in percent} = \eta = \frac{\text{Output Power}}{\text{Input Power}} \times 100 \quad \text{Eq. 2.31}$$

Where,

$\eta$  (Eta) is a universal symbol for efficiency

In electrical engineering, when energy is the subject of analysis, efficiency is defined as follows:

$$\text{Efficiency in percent} = \eta = \frac{\text{Output Energy}}{\text{Input Energy}} \times 100 \quad \text{Eq. 2.32}$$

Although work is not used as commonly in the computation of efficiency in the electromechanical realm, where applicable, the overall system efficiency calculation based on work could be stated as follows:

$$\text{Efficiency (\%)} = \eta = \frac{\text{Work Performed By the Electromechanical System}}{\text{Input Energy}} \times 100$$

**Eq. 2.33**

As obvious from the definitions of efficiency above, since energy cannot be created, efficiency cannot exceed “1” or 100%. The decimal result for efficiency is often converted to, and stated as, a percentage value.

In the following section, we will explore the relationship between power and efficiency in steam, mechanical and electrical systems, and develop better understanding of the flow of power in steam type electrical power generating systems.

### **Power Conversion from Steam to Electrical Form**

The power delivered by steam to the turbine blades, call it  $P_{\text{Steam}}$ , in a simplified - no heat loss, no kinetic head loss, no potential head loss and zero frictional head loss - scenario can be represented by the mathematical relationship stated in form of **Eq. 2.34**. In the context of flow of energy from steam to electricity, functional relationship between electrical power,  $P_{\text{Electrical}}$ , generator efficiency  $\eta_{\text{Generator}}$ , steam turbine efficiency  $\eta_{\text{Turbine}}$ , and  $P_{\text{Steam}}$  can be expressed in form of **Eq. 2.35**.

$${}^1P_{\text{Steam}} = (h_i - h_f) . \quad \text{Eq. 2.34}$$

$${}^2P_{\text{Electrical}} = (P_{\text{Steam}}) \cdot (\eta_{\text{Turbine}}) \cdot (\eta_{\text{Generator}}) \quad \text{Eq. 2.35}$$

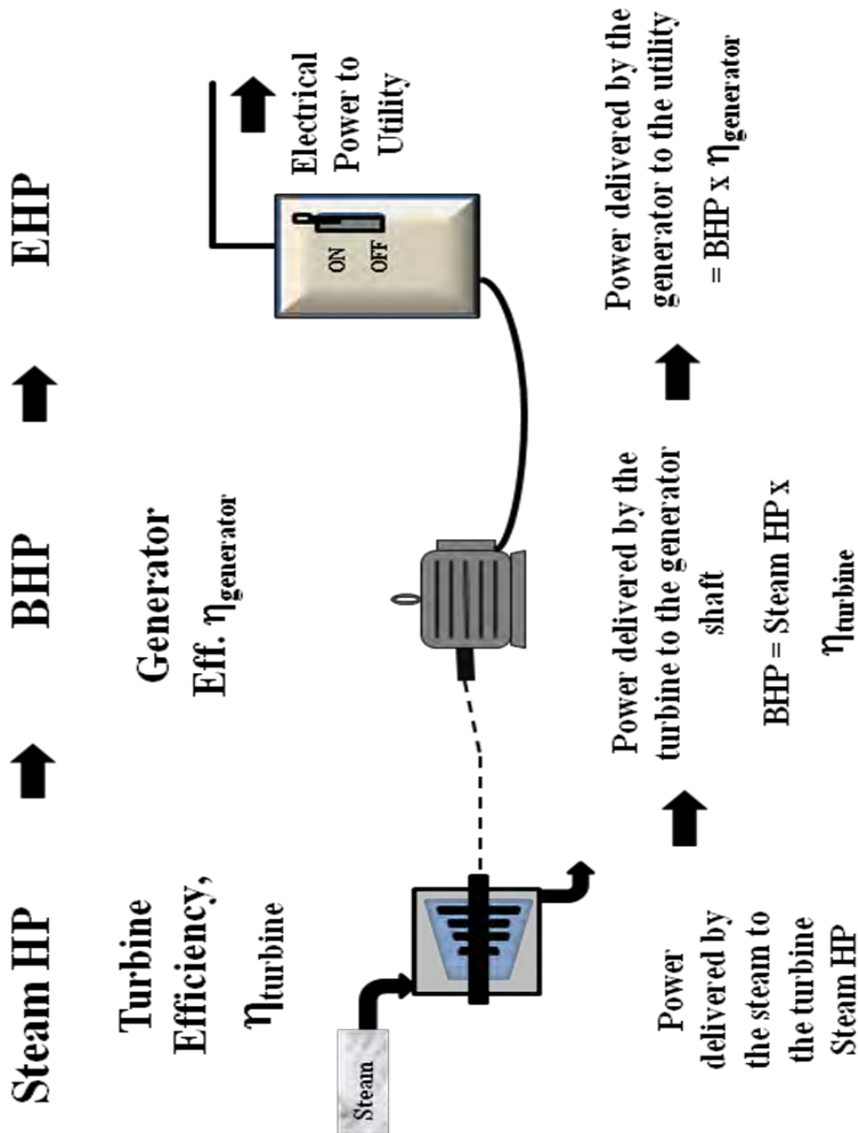
See the power flow diagram depicted in Figure 2.1. Even though this diagram refers to the flow of power, in conformance with Eq. 2.32, it applies just as well to flow of energy. This diagram shows the harnessing of power and energy contained in superheated steam – with high enthalpy or heat content – and their conversion to mechanical brake horsepower in the steam turbine. The brake horsepower thus imparted onto the turbine shaft is conveyed to the

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<sup>1,2</sup>Thermodynamics Made Simple for Energy Engineers, By: S. Bobby Rauf

electrical power generator typically through direct coupling of the turbine and generator shafts. The generator, subsequently, converts the brake horsepower and mechanical energy into electrical power (Watts, kW, MW, etc.) and energy (Wh, KWh, MWh, etc.).

However, as annotated in Figure 2.1, the transformation of power and energy from steam to electricity is de-rated first in the turbine and later in the generator by respective efficiencies.



**Figure 2.1** Steam to Wire Power and Energy Flow – Steam Power Generation System.

The flow of power and energy from - steam to electricity - as depicted in Figure 2.1, is also referred to as “steam to wire” flow of power and energy. Of course, the electrical power galvanized by the generator is presented to the power distribution grid via necessary switchgear and transformers.

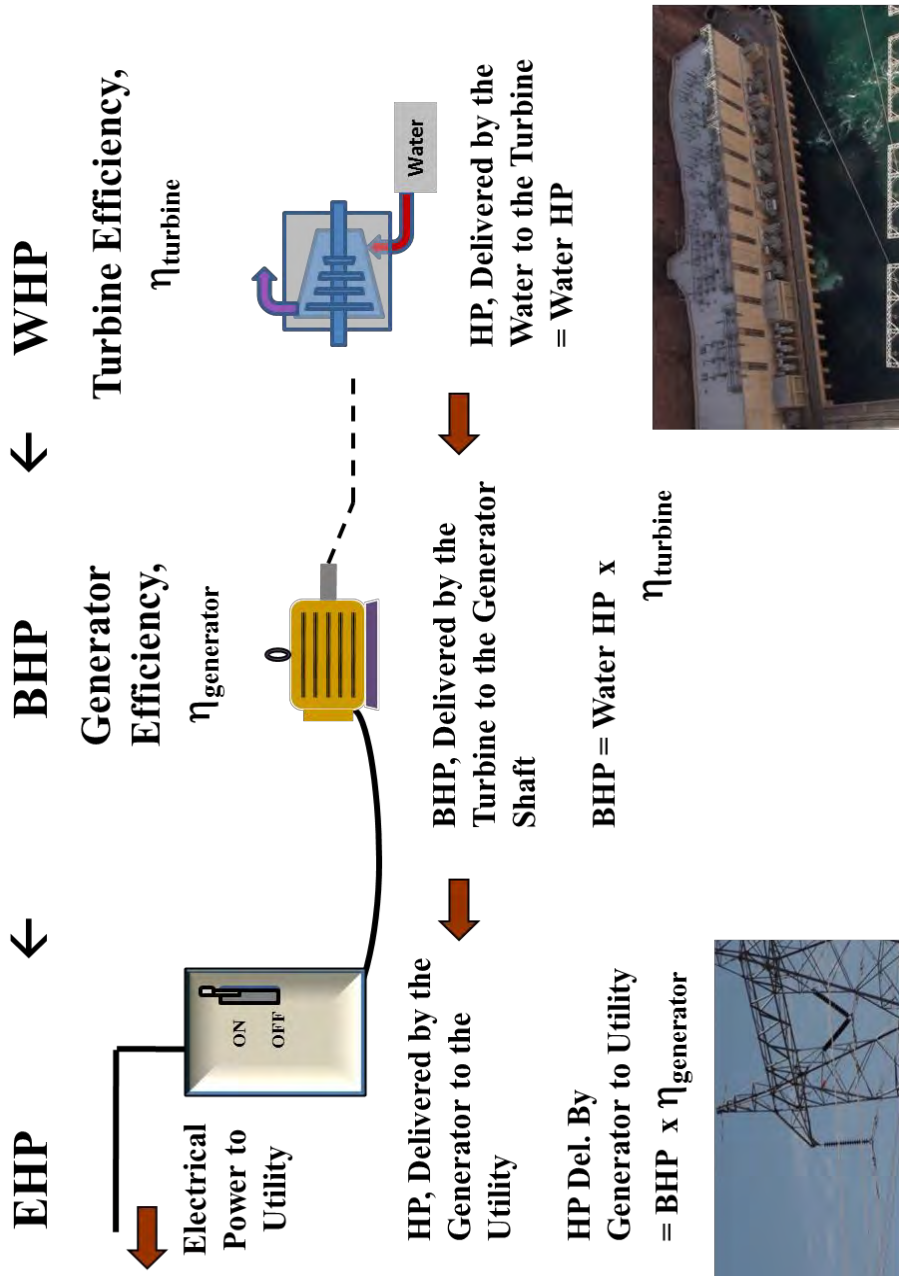


Figure 2.2 – Water to wire power and energy flow.



## Power Conversion from Water to Electrical Form

In hydroelectric power plants, power and energy is transferred from water to electrical form, via hydraulic turbines. This flow of power and energy - referred to as “**Water to Wire**” flow of power - is illustrated in Figure 2.2. The power carried by the water to the turbine blades is called “**Water Horse Power**,” or **WHP**; sometimes, also referred to as *hydraulic horsepower* or *fluid horse power*. As explained in the next section, when energy or power flows from electrical power source to the fluid, through a hydraulic pump, the **WHP** is considered equivalent to, or referred to as, **P<sub>P</sub>**, the *pump horse power*.

The transformation of power from hydraulic form to electrical can be computed through equations similar to the ones listed below:

$$\text{WHP} = \frac{(h_A)(\gamma)(\dot{V})}{550} \quad \text{Eq. 2.36}$$

$$\text{WHP} = \frac{(\Delta P)(\dot{V})}{550} \quad \text{Eq. 2.37}$$

Where,

**WHP** = Water horse power, imparted the water onto the turbine, in hp.

**h<sub>A</sub>** = Net head added to the water, in ft, by virtue of the height of water in the reservoir.

**γ** = Specific weight or weight density = 62.4 lbf/ft<sup>3</sup>

**ΔP** = Differential pressure, across the turbine, in pounds per ft<sup>2</sup>

**$\dot{V}$**  = Volumetric flow rate of water flowing through the turbine,  
measured in ft<sup>3</sup>/sec

In the context of flow of energy from water to electricity, functional relationship between electrical power, **P<sub>Electrical</sub>**, generator efficiency **η<sub>Generator</sub>**, turbine efficiency **η<sub>Turbine</sub>**, and **WHP** can be expressed in form of Eq. 2.38.

$$\mathbf{P_{Electrical}} = (\mathbf{WHP}) \cdot (\boldsymbol{\eta_{Turbine}}) \cdot (\boldsymbol{\eta_{Generator}}) \quad \text{Eq. 2.38}$$

See the power flow diagram depicted in Figure 2.2. Even though this diagram refers to the flow of power, in conformance with Eq. 2.38, it applies just as well to flow of energy. This diagram shows the harnessing of power and energy contained in water – including potential head, kinetic head pressure heads – and their conversion to mechanical brake horsepower in the turbine. The brake horsepower thus imparted onto the turbine shaft is conveyed to the electrical power generator typically through direct coupling of the turbine and generator shafts. The generator subsequently converts the brake horsepower and mechanical energy into electrical power (Watts, kW, MW, etc.) and energy (Wh, KWh, MWh, etc.).

However, as annotated in Figure 2.2 and supported by Eq. 2.38, the transformation of power and energy from water to electricity is first depreciated in the turbine and later in the generator by respective efficiency values of the turbine and generator.

The flow of power and energy from - water to electricity - as depicted in Figure 2.2, is also referred to as “*water to wire*” flow of power and energy. Electrical power generated by the generator is routed to the power grid through the necessary switchgear and transformers.

### **Power Conversion in Hydraulic Pump Systems – Wire to Water Transformation of Power and Energy**

Hydraulic pumps driven by electric motors represent the exact converse of the hydroelectric power generating systems. The flow of power in electric pump applications follows the path depicted in Figure 2.3. Equations 4.36 and 4.37 are still, mathematically, pertinent. However, as annotated in form of Equations 4.39 and 4.40, the WHP equations are interpreted in a context that is opposite of Equations 4.36 and 4.37.

WHP = Fluid horse power delivered by the pump to the water

$$\begin{aligned} &= \frac{(h_A)(\gamma)\left(\dot{V}\right)}{550} \end{aligned} \qquad \text{Eq. 2.39}$$

WHP = Fluid horse power delivered by the pump to the water

$$= \frac{(\Delta P)(\dot{V})}{550} \quad \text{Eq. 2.40}$$

$$\text{WHP} = (P_{\text{Electrical}}) \cdot (\eta_{\text{Motor}}) \cdot (\eta_{\text{Pump}}) \quad \text{Eq. 2.41}$$

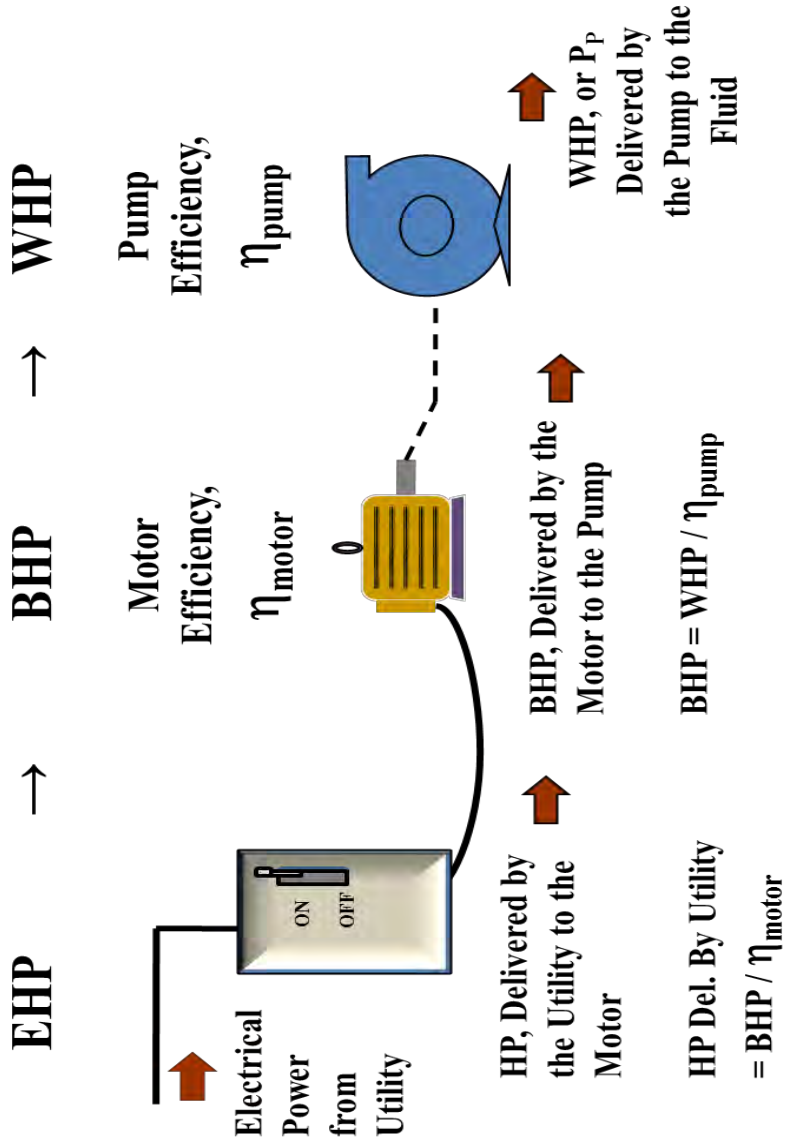


Figure 2.3: Wire to Water Power Flow – Hydraulic Pump System  
 Discussion on Equations 4.39, 4.40 and 4.41, contd:

Where,

**WHP = P<sub>P</sub>** = Water horse power imparted by the pump and motor system onto the water, measured in hp. Also, it is referred to as “pump horse power” or “fluid horse power.”

**h<sub>A</sub>** = Net head added by the pump and motor system onto the water, measured in ft.

**γ** = Specific weight or weight density = 62.4 lbf/ft<sup>3</sup>.

**ΔP** = Differential pressure, across the pump, measured in pounds per ft<sup>2</sup>.

**$\dot{V}$**  = Volumetric flow rate of water through the motor and pump system, measured in ft<sup>3</sup>/sec.

**P<sub>Electrical</sub>** = Electrical power drawn by the motor, from the power source or utility.

**η<sub>Motor</sub>**, = Motor efficiency

**η<sub>Pump</sub>** = Pump efficiency

As evident from Figure 2.3, Eq. 2.39, Eq. 2.40 and Eq. 2.41, in electric motor and hydraulic pump systems, power and energy are transferred from an electrical power source to a hydraulic or fluid system. Because in this scenario the energy stems from the power grid – transported by power transmission wires - and terminates into the fluid, it is referred to as *wire to water* flow of energy and power.

In electric motor and pump systems - as annotated in Figure 2.3 and supported by Eq. 2.41 - during the transformation of power and energy from electricity to water, energy is first depreciated in the motor and later in the pump by respective efficiency values of the motor and the pump.

Some of the efficiency, energy flow and power flow principles and methods described above are illustrated through Examples 2.7 and 2.8 and self-assessment problems at the end of the segment.

### **Example 2.7**

As an energy engineer, you are charged with the task to estimate the heat content or enthalpy,  $h_i$ , of the superheated steam that must be fed to a steam turbine in order to supply **10 MW** (MegaWatt) of electrical power to the electrical grid. Assume that there is no heat loss in the turbine system and that difference between the enthalpies on the entrance and exit ends of the turbine is converted completely into work, minus the inefficiency of the turbine. All of the data available and pertinent to this project is listed below:

- Electrical Power Generator Efficiency: **90%**
- Steam Turbine Efficiency: **70%**
- Mass flow rate for steam,  $\dot{m}$ : **25 kg/s (55 lbm/s)**
- Estimated exit enthalpy,  $h_f$ , of the steam: **2875 kJ/kg (1239 BTU/lbm)**

### **Solution**

Strategy: In order to determine the estimated enthalpy,  $h_i$ , of the incoming steam, we need to start with the stated output (**10 MW**) of the generator and work our way upstream to derive the energy delivered to the vanes of the turbine. The assumption that there is no heat loss in the turbine system and that the difference between the enthalpies on the entrance and exit ends of the turbine is converted completely into work, minus the inefficiency of the turbine, implies that the energy delivered by the steam is equal to the net energy delivered to the turbine vanes. Also, note that net energy delivered to the turbine vanes is reduced or de-rated according to the given efficiency of the turbine.

#### **Solution in SI/Metric Units**

Since,  $1\text{J/s} = 1\text{W}$  and  $1\text{kJ/s} = 1\text{kW}$ ,

$$\begin{aligned}\text{Power output of the generator} &= 10\text{ MW} = 10,000\text{kW} \\ &= \mathbf{10,000\text{kJ/s}}\end{aligned}$$

Brake horsepower delivered by the turbine to the generator, through the turbine shaft is determined as follows:

$$\begin{aligned} \text{BHP} &= \frac{\text{Generator Output}}{\text{Generator Efficiency}} \\ &= \frac{10,000 \text{kJ/s}}{0.9} = 1.11 \times 10^4 \text{kJ/s or } 11,111 \text{kJ/s} \end{aligned}$$

Power delivered by the steam to the turbine vanes is determined as follows:

$$\begin{aligned} P_{\text{steam}} &= \frac{\text{BHP}}{\text{Turbine Efficiency}} \\ &= \frac{1.11 \times 10^4 \text{kJ/s}}{0.7} = 1.5873 \times 10^4 \text{kJ/s or } 15,873 \text{kJ/s} \end{aligned}$$

Of course, we could obtain the same result, in one step, by rearranging and applying Eq. 2.35 as follows:

$$P_{\text{steam}} = \frac{P_{\text{Electrical}}}{(\eta_{\text{Turbine}}) \cdot (\eta_{\text{Generator}})} = \frac{10,000 \text{kJ/s}}{(0.7) \cdot (0.9)} = 15,873 \text{kJ/s}$$

Since the difference in the turbine entrance and exit enthalpies, in this scenario, is equal to the energy delivered to the turbine vanes:

$$P_{\text{steam}} = (h_i - h_f) \cdot \dot{m} \quad \text{Eq. 2.34}$$

$$P_{\text{steam}} = 15,873 \text{kJ/s} = (h_i - 2875 \text{kJ/kg}) \cdot 25 \text{kg/s}$$

$$h_i = \frac{15,873 \text{kJ/s}}{25 \text{kg/s}} + 2875 \text{kJ/kg}$$

$$h_i = 3,509 \text{kJ/kg}$$

#### **Solution in US/Imperial Units**

Power output of the generator = 10 MW = 10,000kW = 10,000kJ/s

Since 1.055 kJ = 1.0 BTU,

$$\begin{aligned} \text{Power output of the generator} &= (10,000 \text{kJ/s}) \cdot (1/1.055 \text{kJ/BTU}) \\ &= 9,479 \text{BTU/s} \end{aligned}$$

Brake horsepower delivered by the turbine to the generator:

$$\text{BHP} = \frac{\text{Generator Output}}{\text{Generator Efficiency}} = \frac{9,479 \text{ BTU/s}}{0.9} = \mathbf{10,532 \text{ BTU/s}}$$

Power delivered by the steam to the turbine vanes is determined as follows:

$$P_{\text{steam}} = \frac{\text{BHP}}{\text{Turbine Efficiency}} = \frac{10,532 \text{ BTU/s}}{0.7} = \mathbf{15,046 \text{ BTU/s}}$$

Since the difference in the turbine entrance and exit enthalpies, once again, is equal to the energy delivered to the turbine vanes:

$$P_{\text{steam}} = (h_i - h_f) \cdot \dot{m} \quad \text{Eq. 2.34}$$

$$15,046 \text{ BTU/s} = (h_i - 1239 \text{ BTU/lbm}) \cdot (55 \text{ lbm/s})$$

$$\begin{aligned} h_i &= \frac{15,046 \text{ BTU/s}}{55 \text{ lbm/s}} + 1239 \text{ BTU/lbm} \\ &= \mathbf{1512 \text{ BTU/lbm}} \end{aligned}$$

**Verification:** The accuracy of the initial enthalpy,  $h_i$ , value of **1512 BTU/lbm** can be verified by simple unit conversion to the metric form as follows:

Since  $1.055 \text{ kJ} = 1.0 \text{ Btu}$ , or, there are  $0.9479 \text{ Btu per kJ}$

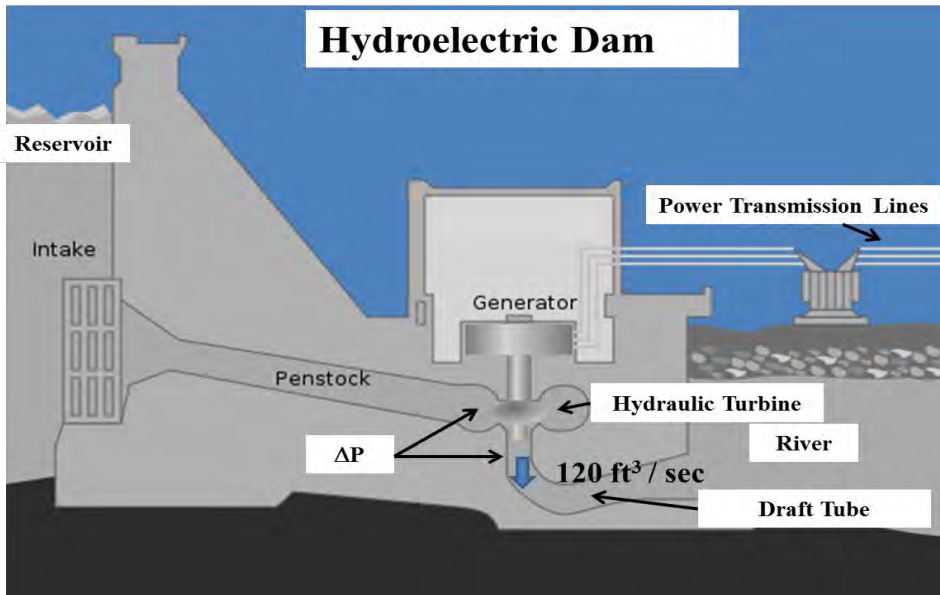
$$\begin{aligned} \frac{1512 \text{ Btu/lbm}}{0.9479 \text{ Btu/kJ}} \times 2.2046 \text{ lb/kg} &\Rightarrow 3,517 \text{ kJ/kg} \\ &\approx 3,509 \text{ kJ/kg (as computed in SI units at the outset)} \end{aligned}$$

### **Example 2.8**

Pressures on the intake and exit ends of a turbine are measured to be 50 psia and 10 psia, respectively. The volumetric flow rate for the fluid (water) is  $120 \text{ ft}^3/\text{sec}$ . See the diagram below. The turbine is driving an electric generator with nameplate efficiency of 90%. Calculate the following:

- a) Water Horsepower delivered by the water to the turbine.
- b) Brake Horsepower delivered by the turbine to the shaft driving the generator. The efficiency of the turbine is 65%.

- c) Maximum power generated, in KW's, by this hydroelectric power generating system.



**Solution**

**Given:**

$$P_{in} = 50 \text{ psia}$$

$$P_{out} = 10 \text{ psia}$$

$$\Delta P = 40 \text{ psia}$$

$$\Delta P \text{ Conv. to psf} = 40 \text{ lb/in}^2 \times 144 \text{ in}^2 / \text{ft}^2 = 5,760 \text{ psf}$$

$$\dot{V} = \text{Volumetric Flow Rate} = 120 \text{ cu - ft / sec}$$

$$\text{Turbine Efficiency} = 65 \%$$

$$\text{Generator Efficiency} = 90 \%$$

- a) According to Eq. 2.32, Water Horsepower delivered by the water to the turbine can be stated as:

$$\text{WHP} = P_p = \frac{(\Delta P) \cdot (\dot{V})}{550}, \text{ provided the differential pressure, } \Delta P, \text{ is expressed}$$

in psf, or pounds per square foot

$$\therefore P_p = \frac{(5760 \text{ lbf / ft}^2) \cdot (120 \text{ ft}^3 / \text{sec})}{550} = 1257 \text{ hp}$$



b) Brake Horsepower delivered by the turbine to the generator – via the shaft  
– at a turbine efficiency,  $\eta_t$ , of 65%:

$$\begin{aligned}\text{BHP} &= \text{WHP} \times \eta_t \\ &= 1257 \text{ hp} \times 0.65 \\ &= \mathbf{817 \text{ hp}}\end{aligned}$$

c) Maximum power generated, in kW's, by this hydroelectric power  
generating system, with the generator efficiency,  $\eta_g$ , of 90%:

$$\begin{aligned}\text{Electrical Power Generated (}^1\text{EHP), in kW} &= \text{BHP} \times \eta_g \times 0.746 \text{ kW / hp} \\ &= 817 \text{ hp} \times 0.9 \times 0.746 \text{ kW / hp} \\ &= 549\text{kW}\end{aligned}$$

---

<sup>1</sup>

Electric Horse Power, abbreviated as EHP, is loosely used to refer to generated electrical power measured in Horse Power, hp or kW.

**Example 2.9**

A pump is to be installed to supply a maximum of 200 ft<sup>3</sup>/sec at a gage pressure ( $\Delta P$ ) of 8,000 psf. Determine the size of the motor required and the size (kVA) of the transformer for this system. Assume that the efficiency of the pump is 85% and that the nameplate efficiency of the “custom” designed special motor is 92%. Power factor of the transformer is 0.8. The frictional and other hydraulic losses are negligible.

**Solution**

Solution strategy in this case would be to use Eq. 2.32 to compute the WHP. Then, the amounts of real power “P” drawn by the motor and delivered by the motor would be computed based on the given efficiencies of the pump and the motor. Once the real power drawn by the motor is computed, we can use the power factor of the transformer to compute the kVA, or apparent power rating “S,” of the transformer.

**Given:**

$$\Delta P = 8000 \text{ psf}$$

$$V = \text{Volumetric Flow Rate} = 200 \text{ cu-ft / sec}$$

$$\text{Pump Efficiency} = 85 \%$$

$$\text{Motor Efficiency} = 92 \%$$

$$\text{Power factor of the transformer} = 0.8$$

According to Eq. 2.32, Water Horsepower delivered by the pump to the water can be stated as:

$$\text{WHP} = P_p = \frac{(\Delta P) \cdot (\dot{V})}{550}, \text{ provided the differential pressure, } \Delta P, \text{ is expressed}$$

in psf, or pounds per square foot

$$\therefore P_p = \frac{(8000 \text{ lbf / ft}^2) \cdot (200 \text{ ft}^3 / \text{sec})}{550} = 2,909 \text{ hp}$$

According to the Wire to Water (hydraulic pump) power flow diagram in Figure 2.3:

$$\begin{aligned} \text{BHP delivered by the motor to the pump must be} &= \frac{\text{WHP}}{\eta_p} \\ &= \frac{2909}{0.85} \text{ hp} = 3,422 \text{ hp} \text{ \{Custom, non-standard, size\}} \end{aligned}$$

∴ the size of this custom motor would be 3,422 **hp**.

Once again, according to the Wire to Water power flow diagram in Figure 2.3:

$$\begin{aligned} \text{EHP drawn by the motor, from the transformer} &= \frac{\text{BHP}}{\eta_{\text{motor}}} \\ &= \frac{3,422}{0.92} \text{ hp} = 3,720 \text{ hp} \end{aligned}$$

$$\text{Or, EHP, in kW} = (3,720 \text{ hp}) \cdot (0.746 \text{ kW/hp}) = 2,775 \text{ kW}$$

Then, according to Eq. 2.23, the apparent power, **S** (kVA) rating of the feeder transformer would be:

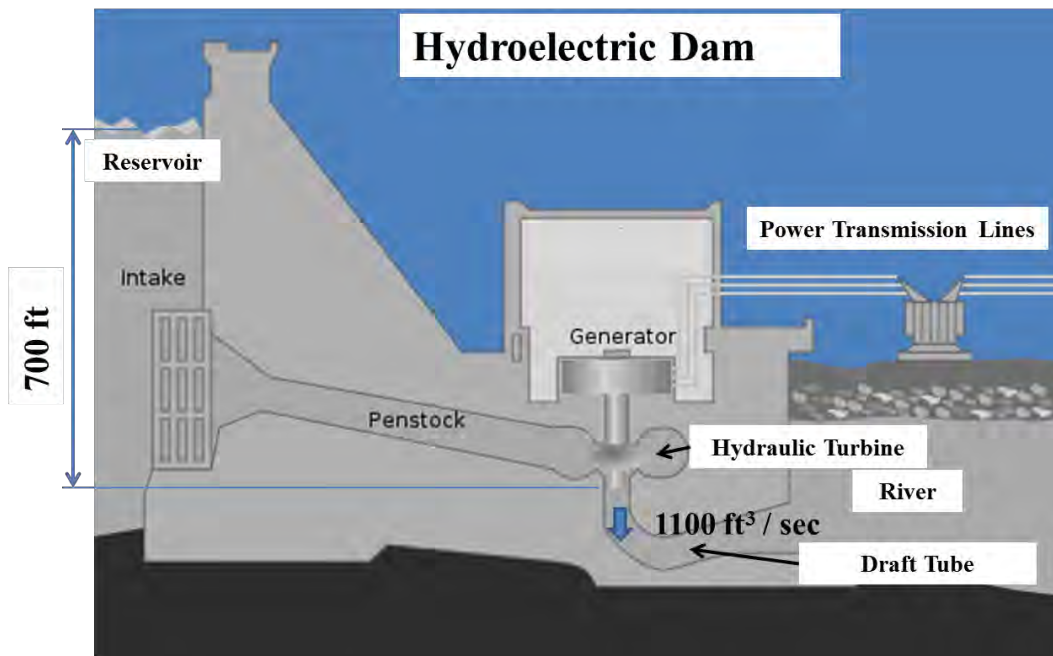
$$\text{PF} = \text{Power Factor} = \frac{|P|}{|S|}$$

$$\text{Or, } |S| = \frac{|P|}{\text{PF}} = \frac{2,775 \text{ kW}}{0.8} = 3,469 \text{ kVA}$$

## Self-assessment Problems & Questions - Segment 2

1. Consider a hydroelectric reservoir where water is flowing through the turbine at the rate of  $1100 \text{ ft}^3 / \text{sec}$ . See the diagram below. The turbine exit point is 700 ft lower than the elevation of the reservoir water surface. The turbine efficiency is 90% and the total frictional head loss through the penstock shaft and turbine system is 52 ft.

- Calculate the power output of the turbine in MW's.
- If the efficiency of the Electric Power Generator is 92%, what would the electric power output be for this hydroelectric power generating system?



2. Which of the following two water heaters would cost the least to operate, on annual cost basis, under the given assumptions?

A. Electric Water Heater:

Estimated annual energy required to heat the water: **9000 kWh**

Efficiency: **95%**

Cost Rate: **\$0.10/kWh**

B. Natural Gas Water Heater:

Estimated annual energy required to heat the water: Same as the Electric water heater

Efficiency: **98%**

Cost Rate: **\$10.87/DT**

3. A computer manufacturing company is testing a prototype for the amount of heat it dissipates as wasted energy over a 10-hour period of operation. The computer is powered by a 24V DC power supply and is designed to draw 3A of current. Determine the total energy dissipated in Btu.

4. In response to a significant near miss incident and midair fire on a new commercial jet aircraft, a governmental agency is performing forensic analysis on the type of Lithium Ion aircraft battery suspected to be the root cause. Estimate the amount of **current** involved in the suspected fault on the basis of the following forensic data:

- Total energy released in the catastrophic failure of the battery: **866kJ**
- Estimated duration of fault: **2 seconds**
- Rated voltage of the battery: **3.7V<sub>DC</sub>**

5. A **156Sin377t** sinusoidal voltage is connected across a load consisting of a parallel combination of a **20 Ω** resistor and a **10Ω** capacitive reactance.

(a) Determine the real power dissipated by the resistor.

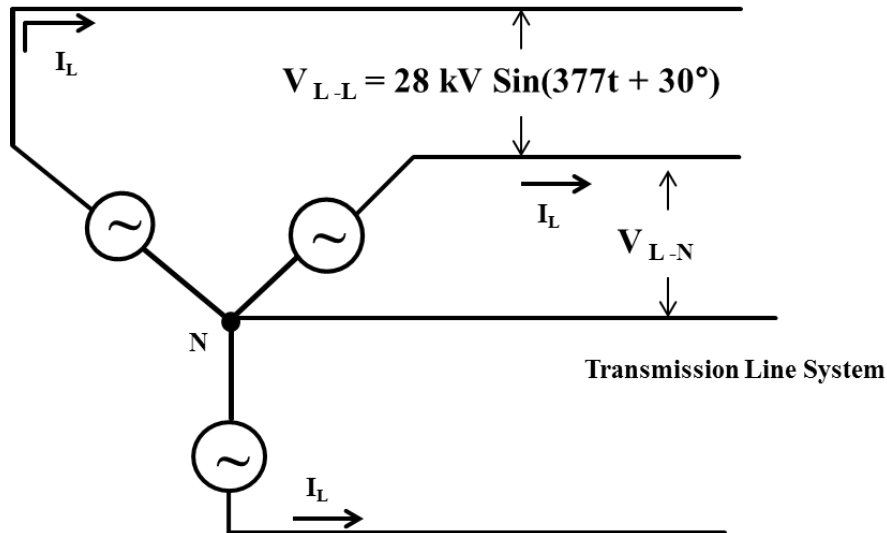
(b) Determine the reactive power stored in a **10Ω** parallel capacitive reactance.

(c) Calculate the total apparent power delivered to this parallel R and X circuit by the AC voltage source.

6. A **156Sin400t** sinusoidal voltage is connected across an unknown resistive load. If the power dissipated in the resistor is 1000 Watts, what is the resistance of the resistive load?

7. The AC circuit shown below depicts a three phase, one-line, schematic of a hydroelectric power generating station. Assume that there is no voltage drop between the generator and the primary side of the transmission system transformer. The line current is indicated by an EMS system to be **10kA**, RMS. Calculate the following if the power factor is known to be **0.95**:

- a) Magnitude of the apparent power presented to the transmission lines.
- b) Magnitude of the real power presented to the transmission lines.
- c) The RMS line to neutral voltage at the source.



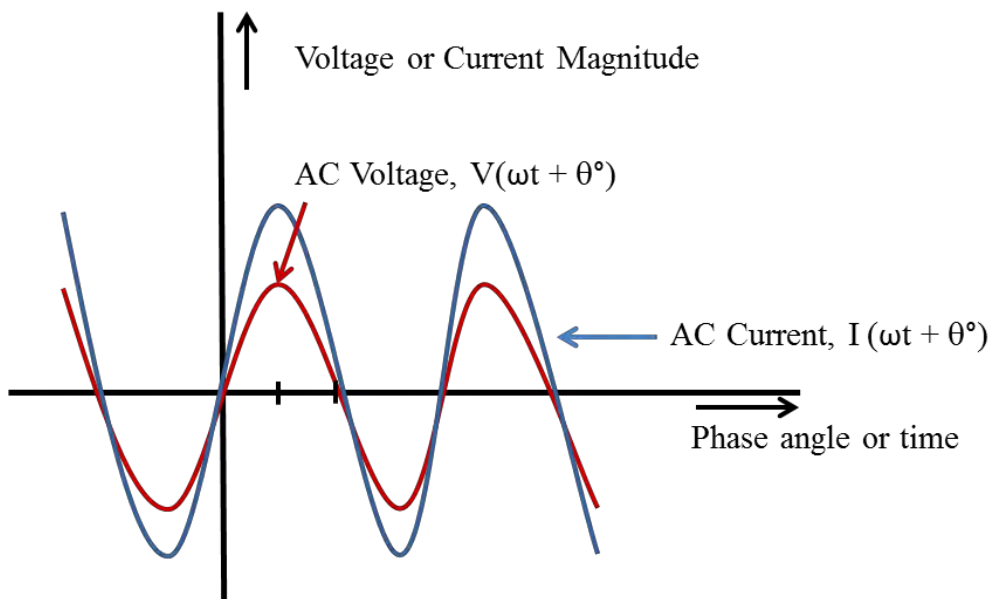
8. A pump is to be installed on the ground floor of a commercial building to supply  $200 \text{ ft}^3/\text{sec}$  of water up to an elevation of 100 ft. Determine the minimum size of the motor for this application. Assume that the efficiency of the pump is 80%. The weight density of water  $\gamma = 62.4 \text{ lbf}/\text{ft}^3$

## Segment 3

### Power Factor

#### Introduction

Power factor was briefly introduced in Segment 2 in order to gain complete and comprehensive understanding of the concepts, principles and application of AC power. Power factor stems from the fact that AC current lags behind or leads the AC voltage in AC circuits or systems that are predominantly inductive. Conversely, AC current leads the AC voltage in AC circuits or systems that are predominantly capacitive. If an AC circuit is purely resistive, or if the inductive reactance is completely offset by the capacitive reactance in the circuit, the voltage and current would be completely coincident, resulting in the perfect “utopian” power factor of “1,” as shown in Figure 3.1.



**Figure 3.1** – Coincident AC voltage and current and perfect power factor.

In AC electrical systems where inductive reactance,  $X_L$ , exceeds  $X_C$ , AC current and voltage will not be coincident. The current will lag behind the voltage with respect to time and phase angle; resulting in a power factor that is less than “1” or 100%. See Figures 3.2 and 3.3. The graph shown in Figure 3.2 shows AC voltage and current plotted as a function of time, while Figure 3.3

depicts AC voltage and current plotted as a function of phase angle. Upon closer examination one can discern the fact that, in both of these graphs, the current function (or graph) *follows* the voltage function in the rise and fall below the horizontal (abscissa) axis. This would constitute a lagging power factor scenario, explained in greater detail in later in this segment. At this point, note that the “*leading*” and “*lagging*” qualifier associated with the term power factor is always assessed from current’s vantage point. In other words, it is always the AC current that is lagging behind, leading or in synch with the AC voltage.

One way to remember that the current lags behind voltage in predominantly inductive circuits and that the current leads the voltage in predominantly capacitive circuits is through a well-known mnemonic:

#### **ELI the ICE man**

- The **L** in **ELI** means *inductance*. The **E** (voltage) comes first, then the **I** (current) lags behind. Inductive reactance produces a lagging power factor
- The **C** in **ICE** means *capacitance*. The **I** (current) comes first (leads) then the **E** (voltage) comes later. Capacitive reactance produces a leading power factor.

In this segment, we will explore the physical aspects of power factor in greater detail, and we will illustrate the difference between lagging power factor and leading power factor. We will explain and demonstrate potential economic benefit that can be derived from improvement of power factor in AC systems. We will explain the consequence of low power factor on the energy productivity<sup>1</sup>, energy cost and the life of electrical equipment. We will conclude the power factor discussion with the introduction of conventional alternatives for power factor improvement.

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<sup>1</sup>Finance and Accounting for Energy Engineers, By S. Bobby Rauf.



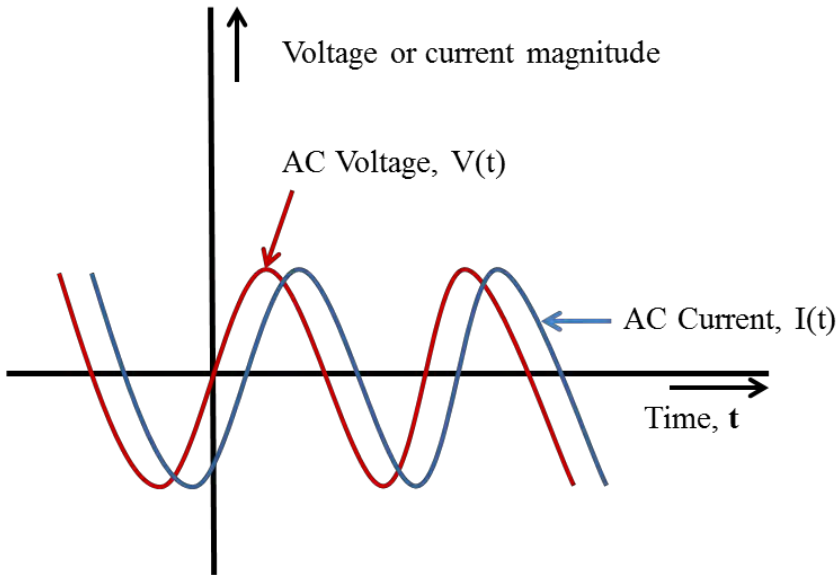


Figure 3.2 – Lagging power factor graph as a function of time.

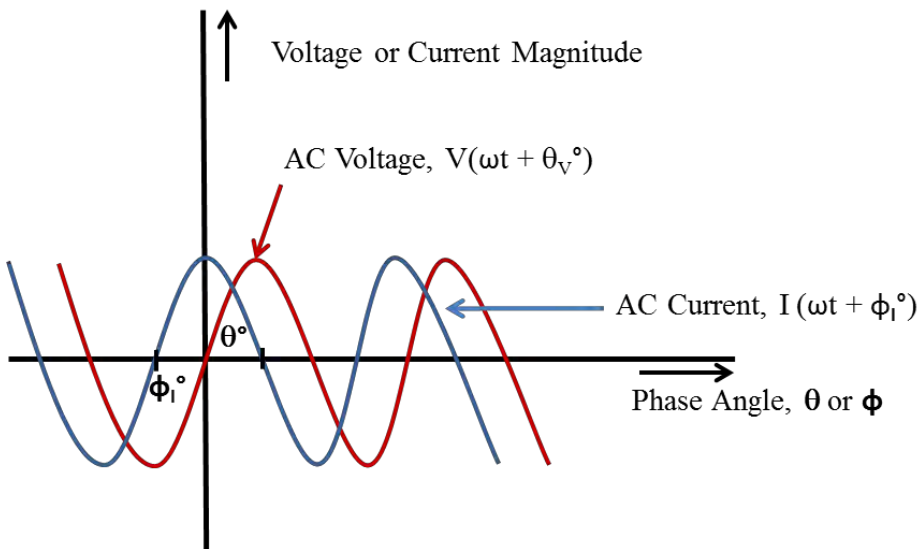


Figure 3.3 – Lagging power factor graph as a function of phase angle.

### Power Factor

Power factor of an AC system may be defined “qualitatively” as the capacity of an AC system to convert transmitted or delivered apparent power to actual

work or other forms of energy. The mathematical or “quantitative” definition of power factor – as introduced in Segment 2 – is represented by Equations 3.1 and 3.2, below.

$$\text{PF} = \text{Power Factor} = \frac{|P|}{|S|} \quad \text{Eq. 3.1}$$

$$\text{PF} = \text{Power Factor} = \text{Cos}(\theta_V - \theta_I) \quad \text{Eq. 3.2}$$

Where,

$|P|$  = Magnitude of the real component of AC apparent power

$|S|$  = Magnitude of the overall AC apparent power

$\theta_V$  = Angle of AC voltage

$\theta_I$  = Angle of AC current, also denoted as  $\phi_I$

### **Lagging Power Factor**

Since loads in most industrial, commercial and institutional facilities consist of inductive equipment such as motors, transformers, solenoids, etc., the inductive reactance,  $X_L$ , in such facilities, exceeds any capacitive reactance,  $X_C$ , that might be present. When AC voltage is applied to such predominantly inductive systems, the resulting AC current lags behind the voltage. If this lag was measured in terms of phase angle, the voltage angle  $\theta_V$  would be greater than the current angle  $\theta_I$ , the angular difference would be positive and would be quantified as  $(\theta_V - \theta_I)$ . The cosine of this angular difference would represent the lagging power factor of the AC system and would be stated in the form of Eq. 3.2. Alternatively, power factor or the “efficiency” of conversion of AC apparent power,  $S$ , to real power  $P$  (rate of performance of work, or delivery of torque and energy) could be represented by the ratio of the magnitudes of the real power  $P$  and apparent power  $S$ , as stipulated by Eq. 3.1.

**Note:** Since cosine of any angle that lies in the 1<sup>st</sup> or 4<sup>th</sup> quadrants of the Cartesian coordinates is positive, power factor would be a positive number - declining with increasing angular difference - whether or not the current lags behind the voltage.

**Example 3.1:**

In an AC system, the voltage angle  $\theta_V = 65^\circ$  and the current angle  $\phi_I = 20^\circ$ .

- a) Calculate and define the power factor
- b) Calculate and define the power factor if the voltage and current angles are reversed.

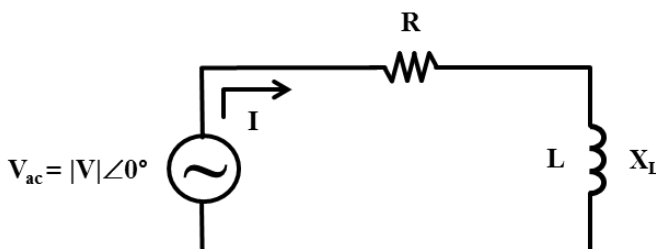
**Solution:**

(a)  $\text{Cos}(65^\circ - 20^\circ) = \text{Cos}(45^\circ) = 0.707$ ; or 70.7% lagging

(b)  $\text{Cos}(20^\circ - 65^\circ) = \text{Cos}(-45^\circ) = \text{Cos}(+45^\circ) = 0.707$ ; or 70.7% leading

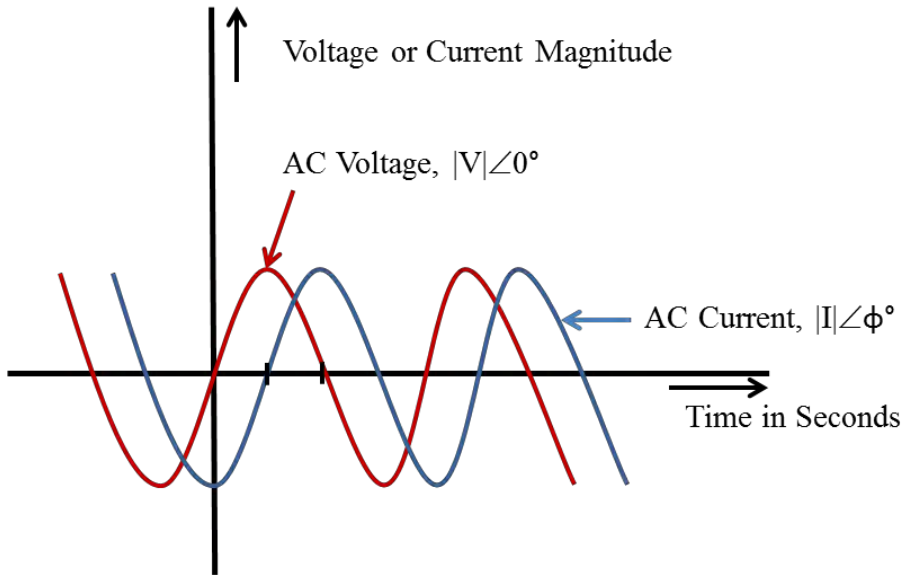
**Lagging Power Factor from Impedance Perspective**

A basic AC circuit with a lagging power factor, as depicted in Figure 3.4, would consist of resistance and inductance, represented by  $R$  and  $X_L$ , respectively. As shown in Figure 3.4, the AC voltage source,  $V_{ac}$ , drives AC current  $I$  through the total resistance  $R$  and the total inductance  $L$  in the AC system. Customarily,  $R$  would represent not only the resistive loads in the overall system but also the inherent resistance of the power distribution system, i.e., resistance of the conductors and other current carrying devices. The total inductance in the system,  $L$ , could be an aggregate of the inductances of all inductors in the system, i.e., motors, solenoids, transformers, etc. The total inductance,  $L$ , could be represented by its inductive reactance  $X_L$ , as shown in Figure 3.4, or by its impedance contribution,  $Z_L$  as annotated in Figure 3.5. While, in practice, there may be some capacitance and capacitive reactance in the system, for the sake of simplicity, it is assumed that this AC system is predominantly inductive,  $X_L \gg X_C$ , or  $X_C$  is negligible.

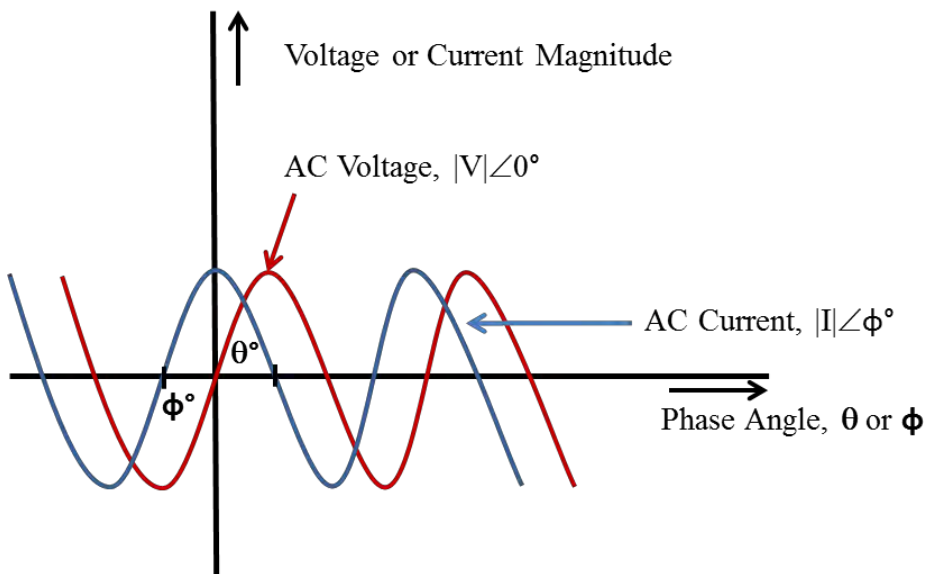


**Figure 3.4** – AC circuit with a lagging power factor

One salient effect of the dominant inductive reactance  $X_L$  in this circuit is that it causes the current to lag behind the voltage. The current lags behind the voltage in terms of *time* as well as the *angle* as depicted graphically in the Figures 3.5 (a) and (b).



a) Current lagging the voltage in time domain.



b) Current lagging the voltage in phase angle domain.

**Figure 3.5** – AC circuit with a lagging power factor; current lagging the voltage in phase angle and time domains.

While Figure 3.4 illustrates the physical – circuit element – aspect of a predominantly inductive AC electrical system and Figure 3.5 highlights the resulting discordance between the voltage and current waveforms, Figure 3.6 helps us visualize the formation of a less-than-perfect, non-unity power factor in terms of vectors.

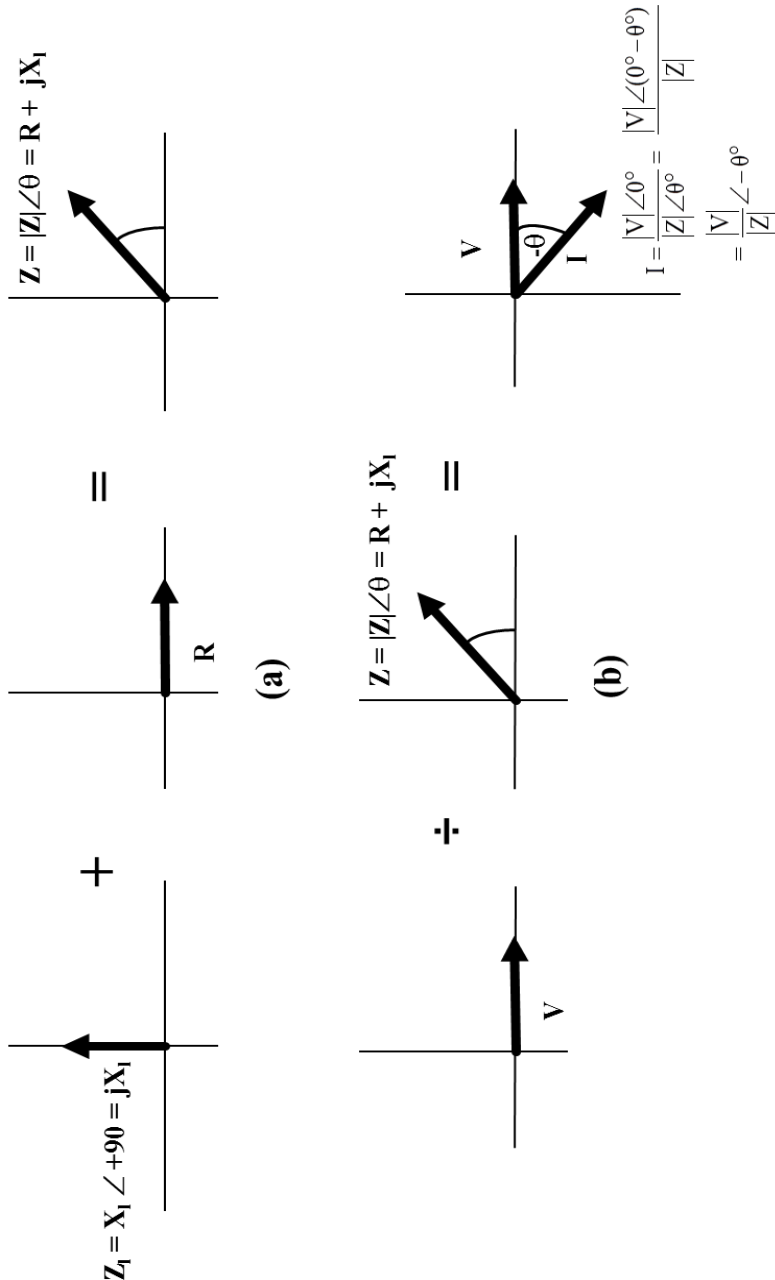


Figure 3.6 – Graphical analysis of lagging power factor

The combination of the resistance and the inductance in the circuit to form the total impedance of the circuit is depicted in form of vectors **R** and **Z<sub>L</sub>** and **Z**, respectively, in Figure 3.6 (a). Note that the impedance contribution of inductance in the circuit is represented by the vector **Z<sub>L</sub>** pointing straight up at an angle of **90°**. As discussed earlier in this text, impedance contribution **Z<sub>L</sub>** by the inductance **L**, or inductive reactance **X<sub>L</sub>** in an AC system, is denoted in complex (rectangular) number form as **jX<sub>L</sub>**. Where, unit vector “**j**” is represented in polar/phasor form as **1∠90°**, and is responsible for assigning the **90°** angle to **Z<sub>L</sub>**.

As always, resistance **R** possesses an angle of zero; therefore, it is represented by a horizontal vector along the x-axis as shown in Figure 3.6 (a).

Vector addition of vectors **Z<sub>L</sub>** and **R** results in the resultant vector **Z**, where, **Z** represents the total or equivalent impedance of the predominantly inductive AC circuit. As ostensible from Figure 3.5 (a), magnitude of **Z**, denoted as **|Z|**, is a geometric hypotenuse of the triangle formed by component vectors **R** and **Z<sub>L</sub>**. Therefore, as annotated in Figure 3.5 (a), **Z** can be represented in polar form as **|Z|∠θ°**, or in rectangular form as **R + jX<sub>L</sub>**.

One of the methods for assessment of power factor requires the knowledge of AC voltage and AC current in polar or phasor form. The voltage is assumed to be at an angle of **0°** and as shown in Figure 3.4, represented in vector/polar form as **|V|∠0°**. The current is not known. The current can be determined by applying ohms law as follows:

$$\begin{aligned} I &= \frac{|V| \angle 0^\circ}{|Z| \angle \theta^\circ} = \frac{|V| \angle (0^\circ - \theta^\circ)}{|Z|} \\ &= \frac{|V|}{|Z|} \angle -\theta^\circ \end{aligned}$$

Then, by definition and in accordance with Eq. 3.2,

$$\text{PF} = \text{Power Factor} = \text{Cos}(\theta_V - \theta_I)$$

Since, in this lagging power factor case,

$$\theta_V = 0^\circ$$

$$\text{And, } \phi_I = \theta_I = -\theta^\circ$$

$$\begin{aligned} \text{PF} = \text{Power Factor} &= \text{Cos}(\theta_V - \theta_I) = \text{Cos}\{0 - (-\theta^\circ)\} \\ &= \text{Cos}(\theta^\circ) \end{aligned}$$

Or, Power Factor = Cosine of the angle of the impedance, Z.

$$\text{PF} = \text{Cos}(\theta_Z) \quad \text{Eq. 3.3}$$

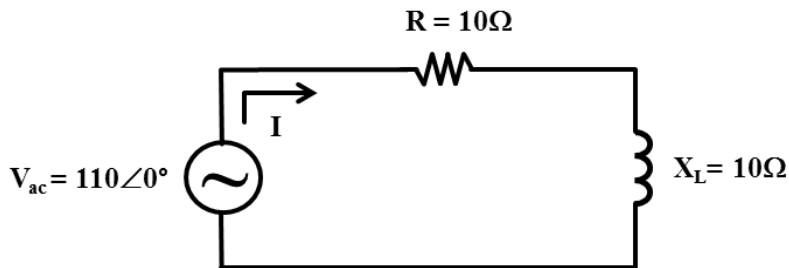
### Example 3.2

A  $110\angle 0^\circ$  voltage is applied to load consisting of a  $10\Omega$  resistance and  $10\Omega$  inductive reactance, in series. Calculate the power factor as seen by the source.

**Ancillary:** Verify the result using Eq. 3.1.

### Solution:

The circuit diagram for this system would appear as follows:



Eq. 3.3, derived above, can be applied to determine the power factor if the angle of the total impedance were known. The total impedance can be assessed, in polar/phasor form, using a scientific calculator or through the angle computation formula covered in the preceding segments.

$$|\theta_Z| = \text{Tan}^{-1}\left(\frac{R}{|X_L|}\right) = \text{Tan}^{-1}\left(\frac{10\Omega}{10\Omega}\right) = \text{Tan}^{-1}(1) = 45^\circ$$

$\therefore$  Power Factor = PF = Cos ( $\theta_Z$ ) = Cos ( $45^\circ$ ) = 0.707,  
or 70.7% lagging.

**Ancillary:** In order to verify the power factor computed above, as obvious from Eq. 3.1, we need to find the real power component **P** and the total apparent power **S** dissipated in the given circuit.

Apparent power **S** can be calculated using Eq. 2.1 introduced in Segment 2.

$$\bar{S} = \bar{V} \cdot \bar{I}^* \quad \text{Eq.2.1}$$

However, we must calculate the phasor value of the AC current first.

$$I = \frac{110 \angle 0^\circ}{|Z| \angle \theta_Z^\circ}$$

$$|Z| = \sqrt{R^2 + X_L^2} = \sqrt{10^2 + 10^2} = \sqrt{200} = 14.14$$

$$\therefore I = \frac{110 \angle 0^\circ}{14.14 \angle 45^\circ} = 7.78 \angle -45^\circ$$

**Note:** The negative angle of the current, with the voltage angle being  $0^\circ$ , signifies the fact that the current is **lagging** behind the voltage, and therefore the power factor will be a **lagging** power factor.

$$S = V \cdot I^* = (110 \angle 0^\circ) \cdot (7.78 \angle 45^\circ) = 856 \angle 45^\circ$$

Note: "\*" stands for conjugation of the original current angle

$$\text{Then, } P = |S| \cdot \cos(45^\circ) = |856| \cdot \cos(45^\circ) = 605 \text{ W}$$

$$\text{And, } Q = |S| \cdot \sin(45^\circ) = |856| \cdot \sin(45^\circ) = 605 \text{ W } \{\text{Redundant}\}$$

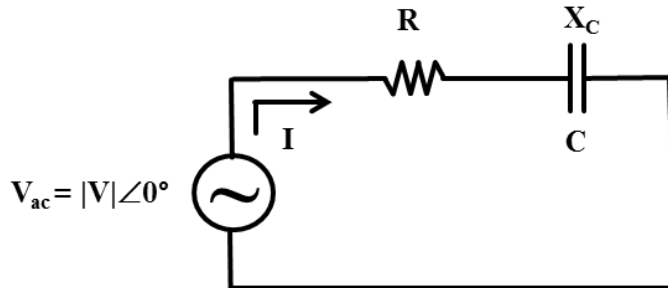
$$\therefore \text{PF} = \frac{P}{|S|} = \frac{605}{856} = 0.707 \text{ or } 70.7\% \text{ lagging.}$$

### Leading Power Factor and Impedance

A basic AC circuit with a leading power factor, as depicted in Figure 3.7, would consist of resistance and capacitance, represented by **R** and **X<sub>C</sub>**, respectively. As shown in Figure 3.7, the AC voltage source, **V<sub>ac</sub>**, drives AC current **I** through the total resistance **R** and the total capacitance **C** in the AC system. The resistance, **R**, would represent not only the resistive loads in the overall system but also the inherent resistance of the power distribution system, i.e., resistance of the conductors and other current carrying devices. The total capacitance in the system, **C**, could be an aggregate of the

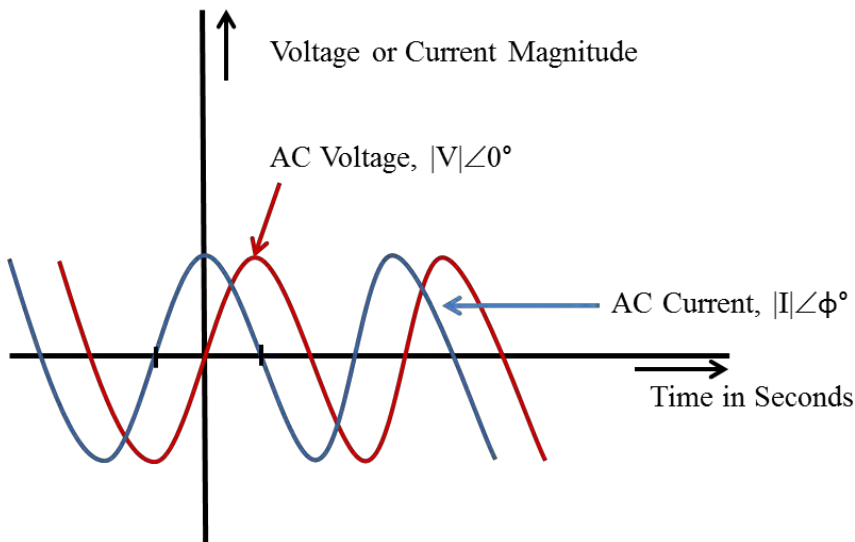


capacitances in the system, transmission line capacitance, etc. Leading power factor can also be caused by machines (i.e. certain power generators) that might be operating at a leading power factor, computer switched mode power supplies, etc. The total capacitance,  $C$ , is represented by its capacitive reactance  $X_C$ , as shown in Figure 3.7, or by its impedance contribution,  $Z_C$  as annotated in Figure 3.9. While, in practice, there may be some inductance and inductive reactance in the system, for the sake of simplicity, it is assumed that this AC system is predominantly capacitive,  $X_C \gg X_L$ , or  $X_L$  is negligible.

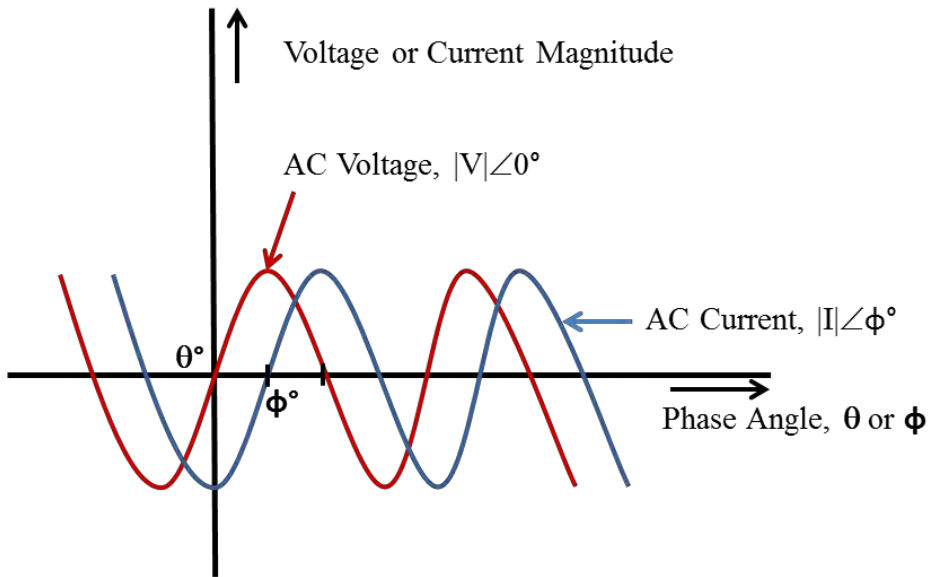


**Figure 3.7** – AC circuit with a leading power factor

One distinct effect of the dominant capacitive reactance  $X_C$  in this circuit is that it causes the current to lead the voltage. The current leads the voltage in terms of *time* as well as the *angle*, as depicted graphically in the Figures 3.8 (a) and (b).



a) Current leading the voltage in time domain.



b) Current leading the voltage in phase angle domain.

**Figure 3.8** – AC circuit with a leading power factor; current leading the voltage in phase angle and time domains

While Figure 3.7 illustrates the physical – circuit element – aspect of a predominantly capacitive AC electrical system and Figure 3.8 highlights the resulting discordance between the voltage and current waveforms, Figure 3.9 depicts the formation of a, less than perfect, non-unity power factor in terms of vectors.

The combination of the resistance and the capacitance in the circuit to form the total impedance of the circuit is depicted in form of vectors **R** and **Z<sub>C</sub>** and **Z**, respectively, in Figure 3.9 (a). Note that the impedance contribution of capacitance in the circuit is represented by the vector **Z<sub>C</sub>** pointing directly downward at an angle of **-90°**. As discussed earlier in this text, impedance contribution **Z<sub>C</sub>** by the capacitance **C** or capacitive reactance **X<sub>C</sub>** in an AC system is denoted in complex (rectangular) number form as **-jX<sub>C</sub>**. Where, unit vector “-j” is represented in polar/phasor form as **1∠-90°**, and is responsible for assigning the **-90°** angle to **Z<sub>C</sub>**.

Resistance **R** portends an angle of zero; therefore, it is represented by a horizontal vector along the x-axis as shown in Figure 3.9 (a).

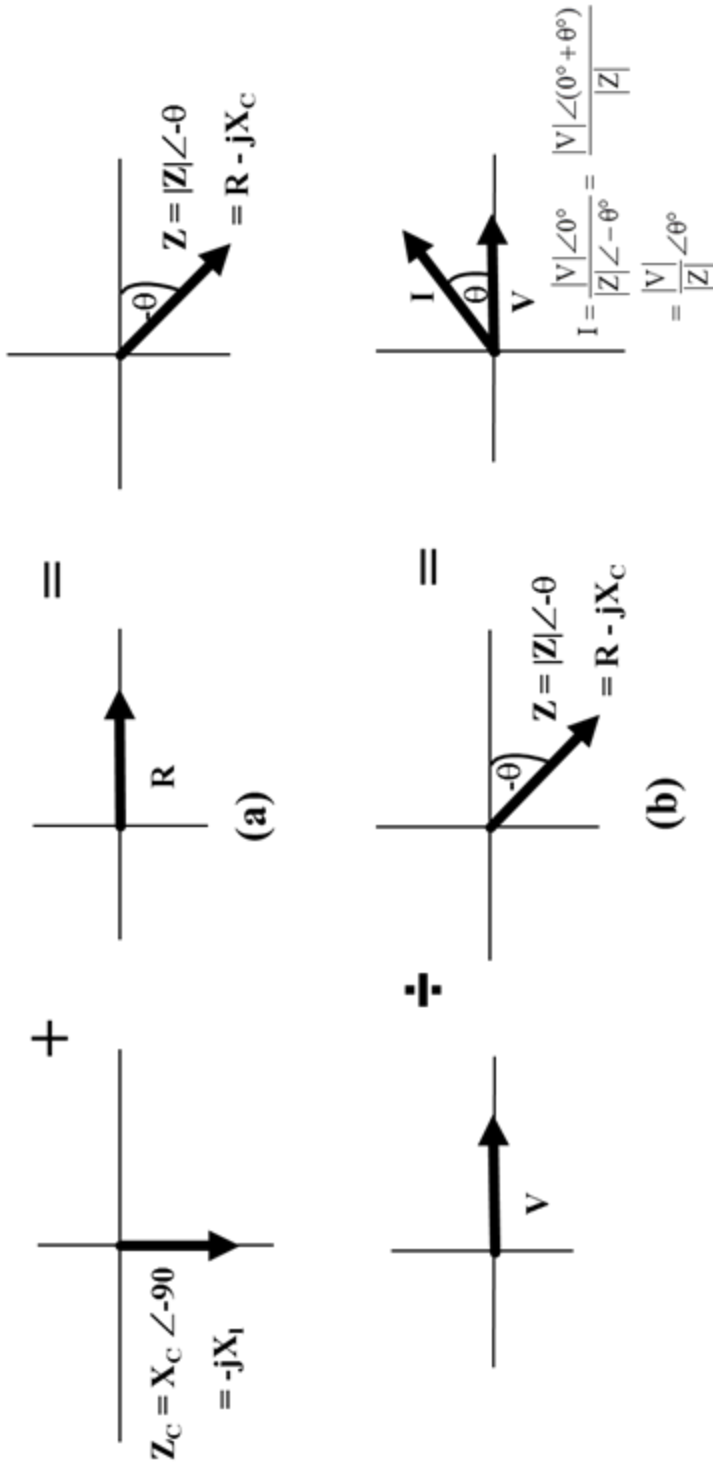


Figure 3.9 – Graphical analysis of leading power factor

Vector addition of vectors  $\mathbf{Z}_C$  and  $\mathbf{R}$  results in the resultant vector  $\mathbf{Z}$ , where,  $\mathbf{Z}$  represents the total or equivalent impedance of the predominantly capacitive AC circuit. As obvious from the Figure 3.9 (a), magnitude of  $\mathbf{Z}$ , denoted as  $|\mathbf{Z}|$ , is a geometric hypotenuse of the triangle formed by component vectors  $\mathbf{R}$  and  $\mathbf{Z}_C$ . Therefore, as annotated in Figure 3.9 (a),  $\mathbf{Z}$  can be represented in polar form as  $|\mathbf{Z}| \angle -\theta^\circ$ , or in rectangular form as  $\mathbf{R} - j\mathbf{X}_C$ .

One approach for assessment of power factor requires the knowledge of AC voltage and AC current in polar or phasor form. The voltage is assumed to be at an angle of  $0^\circ$  and as shown in Figure 3.7, represented in vector/polar form as  $|\mathbf{V}| \angle 0^\circ$ . The current is not known. The current - as with the lagging power factor scenario - can be determined by applying ohms law as follows:

$$\begin{aligned} \mathbf{I} &= \frac{|\mathbf{V}| \angle 0^\circ}{|\mathbf{Z}| \angle -\theta^\circ} = \frac{|\mathbf{V}| \angle (0^\circ + \theta^\circ)}{|\mathbf{Z}|} \\ &= \frac{|\mathbf{V}|}{|\mathbf{Z}|} \angle \theta^\circ \end{aligned}$$

Then, by definition and in accordance with Eq. 3.2,

$$\text{PF} = \text{Power Factor} = \text{Cos}(\theta_V - \theta_I)$$

In this leading power factor case,

$$\theta_V = 0^\circ$$

$$\text{And, } \phi_I = +\theta^\circ, \text{ or just } \theta^\circ$$

$$\begin{aligned} \text{PF} = \text{Power Factor} &= \text{Cos}(\theta_V - \theta_I) = \text{Cos}(0 - \theta^\circ) \\ &= \text{Cos}(-\theta^\circ) = \text{Cos}(\theta^\circ) \end{aligned}$$

Or, Power Factor = Cosine of the angle of the impedance,  $\mathbf{Z}$ .

$$\text{PF} = \text{Cos}(\theta_Z), \text{ which is the same as in the lagging power factor scenario}$$

Eq. 3.3

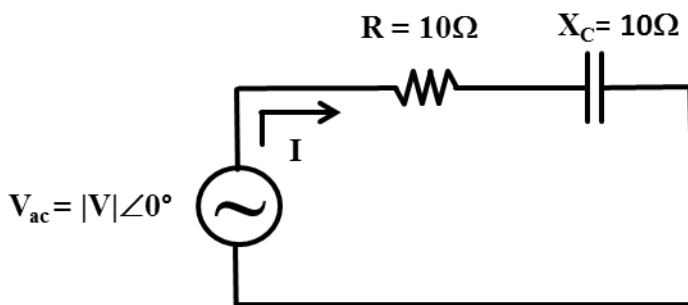
**Example 3.3**

A  $110\angle 0^\circ$  voltage is applied to load consisting of a  $10\Omega$  resistance and  $10\Omega$  capacitive reactance, in series. Calculate the power factor as seen by the source.

**Ancillary:** Verify the result using Eq. 3.1.

**Solution:**

The circuit diagram for this system would appear as follows:



Equation 3.3, derived above, can be applied to determine the power factor if the angle of the total impedance were known. The total impedance can be assessed, in polar/phasor form, using a scientific calculator or through the angle computation formula covered in the preceding segments.

$$|\theta_Z| = \text{Tan}^{-1} \left( \frac{R}{|X_L|} \right) = \text{Tan}^{-1} \left( \frac{10\Omega}{10\Omega} \right) = \text{Tan}^{-1} (1) = 45^\circ$$

As shown in Figure 3.9 (b),  $Z$ , in the leading power factor scenario lies in the fourth quadrant, therefore,  $\theta_Z = -45^\circ$ .

$$\therefore \text{Power Factor} = \text{PF} = \text{Cos} (\theta_Z) = \text{Cos} (- 45^\circ) = 0.707,$$

or 70.7% leading.

**Note:** Because power factor computation is based on “Cosine” of impedance angle, power factor remained the same as we transitioned from  $10\Omega$  inductive

reactance system in Example 3.2 to a  $10\Omega$  capacitive reactance system in Example 3.3; it simply transformed from being lagging to leading.

**Ancillary:** In order to verify the power factor computed above, as obvious from Eq. 3.1, we need to find the real power component **P** and the total apparent power **S** dissipated in the given circuit.

Apparent power **S** can be calculated using Eq. 2.1 introduced in Segment 2.

$$\vec{S} = \vec{V} \cdot \vec{I}^* \quad \text{Eq.2.11}$$

However, as in Example 3.2, we must calculate the phasor value of the AC current first.

$$I = \frac{110\angle 0^\circ}{|Z|\angle -\theta_Z^\circ}$$

$$|Z| = \sqrt{R^2 + X_L^2} = \sqrt{10^2 + 10^2} = \sqrt{200} = 14.14$$

$$\therefore I = \frac{110\angle 0^\circ}{14.14\angle -45^\circ} = 7.78\angle +45^\circ \text{ or } 7.78\angle 45^\circ$$

**Note:** The positive angle of the current, with the voltage angle being  $0^\circ$ , signifies the fact that the current is **leading** the voltage, and therefore the power factor will be a **leading** power factor.

$$S = V \cdot I^* = (110\angle 0^\circ) \cdot (7.78\angle -45^\circ) = 856\angle -45^\circ$$

Note: \* stands for conjugation of the original current angle

$$\text{Then, } P = |S| \cdot \cos(-45^\circ) = |856| \cdot \cos(-45^\circ) = 605\text{W}$$

$$\text{And, } Q = |S| \cdot \sin(-45^\circ) = |856| \cdot \sin(-45^\circ) = -605\text{W} \{\text{Redundant}\}$$

$$\text{Or, } S = P + jQ = 605 + j(-605) = 605 - j605 \text{ VA}$$

$$\therefore \text{PF} = \frac{P}{|S|} = \frac{605}{856} = 0.707 \text{ or } 70.7\% \text{ leading.}$$

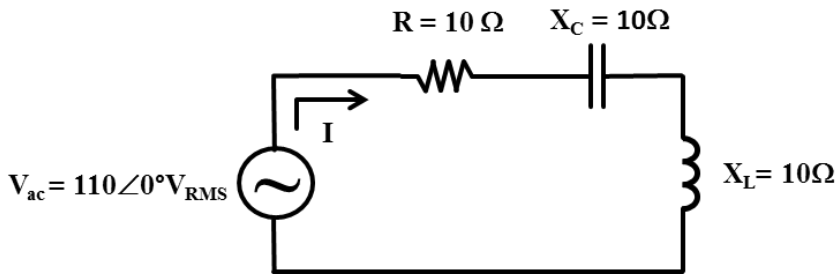
**Example 3.4**

An  $X_L = 10\Omega$  capacitive reactance worth of capacitance is added in series with the series resistor and inductive reactance AC circuit from Example 3.2.

- (a) Draw the circuit diagram for the new circuit.
- (b) Determine the net impedance of the new circuit.
- (c) Calculate the new power factor.

**Solution:**

(a)



(b) Since all elements in the modified circuit are connected in series, the total or net impedance in this AC circuit would be as follows:

$$Z = R + jX_L - jX_C = 10\Omega + j10\Omega - j10\Omega = 10\Omega,$$

or,  $Z = 10\Omega\angle 0^\circ\Omega$

(c) Since, in this case, the inductive reactance is completely offset by the capacitive reactance, the net or total impedance is composed solely of pure resistance of  $10\Omega$ . As discussed earlier in this segment and Segment 2, power factor in circuits that are purely resistive or have “0” net reactance, is “1” or 100%.

This can be proven by applying Eq. 3.3 derived earlier, and recognizing the fact that  $\theta_Z$ , the angle of total impedance  $Z$ , as derived in part (b) is  $0^\circ$ :

Power Factor = Cosine of the angle of the impedance,  $Z$ .

$$\text{Or, PF} = \text{Cos}(\theta_Z) = \text{Cos}(0) = 1$$

As obvious by now, in essence Example 3.4 shows us how capacitive reactance can be added to improve or correct power factor in a predominantly inductive AC circuit. This serves as an appropriate segue into the next section that pertains to the Power Factor Correction.

## **Power Factor Correction**

There are multiple methods for correcting power factor. Some of the more common approaches for correcting power factor are as follows:

- a) Addition of power factor correcting capacitors
- b) Addition of power factor correcting inductors
- c) Frequency manipulation

The power factor correction method chosen depends on the composition or design of the AC system. In general, if the power factor is less than “1,” or 100%, due to the predominance of inductive reactance,  $X_L$ , capacitance or capacitive reactance,  $X_C$ , must be added in series or shunt to raise the power factor closer to 100%. In fact, Example 3.4 – where  $10\Omega$  of  $X_L$  is offset by the addition of  $10\Omega$  of  $X_C$  - is a classic illustration of this approach. Frequency manipulation method is applicable when, in addition to resistance, the both inductive and capacitive reactances are present. As described later in this segment, when the power factor is less than 100% due to the dominance of capacitance in an AC electrical system, inductive reactance may be introduced into the circuit to offset the capacitive reactance and improve the system’s power factor.

### **Addition of capacitance or capacitive reactance**

Since power factor correcting capacitors, essentially, impact the AC circuits or systems through addition of capacitive reactance  $X_C$ , or  $Z_C = -jX_C$ , their addition to a predominantly inductive circuit yields different results depending upon whether they are connected in series with the load or in parallel (or shunt). Example 3.4 represents a scenario where power factor correcting capacitance is added in series. Comprehensive discussion on the distinction between addition of power factor correcting capacitors in parallel versus series is beyond the scope of this text. Recognizing the fact that the difference in parallel versus series addition of capacitors is mainly due to the difference in parallel versus series addition of impedances in general, should suffice.



Power factor correcting capacitors can also be viewed as devices used to reduce current in power systems. Capacitors do so by providing a leading current to loads that require lagging current. In other words, capacitors provide “off- setting” current for inductive loads.

Another side benefit in the use of a series capacitor is in the fact that they can reduce voltage drop, especially in motors starting situations and other high impulse loads. This can be attributed to the fact that capacitors, as discussed earlier in this text, don't permit instantaneous drop or rise in voltage.

There are multiple approaches for assessment of capacitance required for specific improvement of power factor. One method relies on the knowledge of existing and desired reactive powers:  $Q_1$  and  $Q_2$ ; the operating voltage and the frequency, respectively. The formula for determination of the required capacitance is as follows:

$$C = \frac{(Q_1 - Q_2)}{2\pi f V^2} \quad \text{Eq. 3.4}$$

Where,

$C$  = Capacitance, in Farads, required to reduce the reactive power from  $Q_1$  to  $Q_2$

$Q_1$  = Initial, higher, reactive power, in VARs

$Q_2$  = Improved, lower, reactive power, in VARs

$V$  = Voltage, in Volts

$f$  = Frequency, in Hz

This formula is premised on the following mathematical definitions of reactive power  $Q$ :

$$|Q| = \frac{V^2}{X_C} = V^2 2\pi f C \quad \text{Eq. 3.5}$$

Or, for inductive reactance:

$$|Q| = \frac{V^2}{X_L} = \frac{V^2}{2\pi f L} \quad \text{Eq. 3.6}$$

An electrical engineer may use Eq. 3.4 to determine the capacitance required to improve the power factor, by a specific amount, in lieu of the apparent power (S) reduction approach. The apparent power based power factor calculation method is illustrated, later, through Example 3.7.

**Example 3.5**

The HMI (Human Machine Interface) monitor of an EMS system, monitoring a chilled water pump motor, is showing that the motor is drawing a reactive power,  $Q_1$ , of 50kVARs. Funding has been allocated as a part of a DSM (Demand Side Management) energy conservation project to improve the power factor of the motor branch circuit such that the reactive power is reduced to  $Q_2$  of 10kVARs. The branch circuit is operating at 240 V<sub>RMS</sub>. The system frequency is 60 Hz. Calculate the amount of capacitance that must be added to achieve the power factor improvement project objective.

**Solution:**

Apply Eq. 3.4:

$$C = \frac{(Q_1 - Q_2)}{2\pi fV^2} = \frac{(50\text{kVAR} - 10\text{kVAR})}{2\pi(60\text{hz})(240\text{V})^2}$$
$$= \frac{(50,000\text{VAR} - 10,000\text{VAR})}{2\pi(60\text{hz})(240\text{V})^2} = 0.001843\text{F} = 1.84\text{mF}$$

**Example 3.6**

**Power Factor Improvement and Cost Savings:** An air compressor station is consuming 2,000 kW at a power factor of 0.7. The utility company charges a \$5.00/kVA per month as penalty for poor power factor. What would be the annual, pre-tax, savings if capacitors could be installed and power factor improved to 0.9?

**Solution:**

Since the objective is to assess the cost savings on the basis of apparent power S (kVA) reduction, we must determine the apparent power  $S_1$  being drawn by the air compressor motor at the existing power factor of 0.7 (70%), and the apparent power  $S_2$  at the desired power factor of 0.9 (90%). We can compute  $S_1$  and  $S_2$  by rearranging and applying Eq. 3.1.

$$\text{PF} = \text{Power Factor} = \frac{|P|}{|S|} \quad \text{Eq. 3.1}$$

Rearrange Eq. 3.1:

$$\text{Magnitude of Apparent Power } |S| = \frac{|P|}{\text{PF}}$$

Therefore,

$$\begin{aligned} \text{Magnitude of existing apparent power } |S_1| &= \frac{|P|}{\text{PF}_1} = \frac{2,000\text{kW}}{0.7} \\ &= 2857\text{kVA} \end{aligned}$$

$$\begin{aligned} \text{Magnitude of desired apparent power } |S_2| &= \frac{|P|}{\text{PF}_2} = \frac{2,000\text{kW}}{0.9} \\ &= 2222\text{kVA} \end{aligned}$$

Note that real power drawn, which is a function of the actual work performed or torque generated, remains unchanged at 2,000kW.

Annual savings attributed to power factor improvement

$$\begin{aligned} &= (2857\text{kVA} - 2222\text{kVA}) \cdot \left( \frac{\$5}{\text{kVA} \cdot \text{Month}} \right) \cdot (12 \text{ Months/Year}) \\ &= \$38,095 \end{aligned}$$

### **Automatic Switching PF Correcting Capacitor System**

Imagine a plant engineering project that entails installation of a large, 500hp, air compressor. The 500 hp rating of the air compressor is premised on the size of the motor. In other words, the 500 hp rating, in most cases would be real power (**P**) delivered by the motor to the compressor. However, as we discussed earlier, motors add *inductive reactance* to electrical system. So, the branch circuit the motor pertains to, in this facility, would have a non-unity, or less than 100% power factor, say 80%. Sizeable loads, such as large air compressors, therefore, degrade the aggregate power factor of the entire electrical system. On new installations, such as the one being considered here, degradation of power factor can be prevented by including automatic switching power factor correcting capacitor banks such as the one shown in

Figure 3.10. Automatic switching power factor correcting capacitor banks constitute a more optimal and effective approach for automatic introduction and removal of capacitance to maintain the system power factor at an optimal level.

The controls in an automated power factor correcting system connect or disconnect capacitors as the inductive load appears and disappears from the overall electrical circuit. This ensures that the power factor correcting capacitors – and the associated capacitive reactance – drop out of the circuit when the compressor motor load (and associated inductive reactance) switches off. **Important point:** If the power factor capacitors were to stay connected to the compressor motor branch circuit when the motor is off, the circuit would become predominantly capacitive, as  $X_C$  would exceed  $X_L$ , resulting in a low leading power factor. In other words, if the capacitor bank is not disconnected when the compressor based inductive reactance,  $X_L$ , drops out, it would result in a *capacitive reactance “overdose.”*



**Figure 3.10** – Automated power factor correcting capacitor bank.

As annotated in Figure 3.10, there are six key functional components in the pictured automatic power factor correcting system:

1. Control Contactor or Relay: for connecting and disconnecting of the capacitors.
2. Terminal Strip or Connection Points: for interface to the motor circuit, sensors, and other field connections.
3. Slow Blow Fuses: for overload protection.
4. Inrush Limiting Contactors: for limiting the motor inrush current.
5. Capacitors (single-phase or three-phase units, delta-connection: power factor correcting capacitors ( $X_C$ )).
6. Control Transformer: for obtaining control voltage, i.e. 110 VAC, from the existing power source, operating at power circuit voltage, i.e. 480 VAC.

### **General information on power factor and the application of power factor correcting capacitors**

**What is a suitable power factor goal for industrial, commercial and institutional facilities?** It would be difficult to present a specific and perfect answer to this question. While one could find enough data to support the claim that the overall average power factor in the United States is approximately 82%, the best answer varies from situation to situation, and industry to industry. For example, on one end of the spectrum, one might observe power factors approaching 90% to 97% in the institutional and commercial structures, while on the low end of the power factor spectrum it is not uncommon to come across power factors to the tune of 50% or lower in certain foundry, plastics and chemical industries, where the melting processes employ electrical inductive heating.

**What is the financial return and payback associated with power factor correction projects?** Some utilities base demand charges on kVA demanded by the customer facility. In such cases, an approximate 10% demand (kVA) reduction can be realized if a power-factor-correction capacitor bank is installed to improve the power factor from 82% to 90%. Such an investment has a typical payback of, approximately, two (2) years. Note, however, that the payback periods, on power factor correction based energy projects - or for that matter, on any energy project - will be *shorter* for consumers located in

higher energy cost regions, i.e. the North Eastern United States, the Western states and Hawaii. Conversely, the payback periods will be longer in lower energy cost regions, i.e. the South East, the Dakota's, etc. See **Finance and Accounting for Energy Engineers**, by **S. Bobby Rauf**.

**Application, installation, operation and maintenance of power factor correcting capacitors:** Application of power factor correcting capacitors should be preceded by proper analysis and design. Improper application of power factor correcting capacitors can damage the electrical system. Improper application of power factor correcting capacitors can cause fuses to blow and can amplify system harmonics. After installation, capacitor banks must be included in the facility's Preventive Maintenance Program. Loose connections and inherent defects, when ignored, can lead to unanticipated failures, faults, downtimes and arc flash hazards<sup>1</sup>.

Higher power factor means lower apparent power,  $S$  (in kVA), demanded and lower "unproductive" current in the electrical system. This "unproductive" current is wasted as heat, as it flows through conductors, protective devices, power monitoring devices, transformers and loads. As we know in general, operation of equipment above normal temperatures results in shortening of the life of equipment. Case in point and analogy: overheated machines, such as automobile engines, tend to cease and fail when overloaded or operated at abnormally high temperatures. Therefore, improving the power factor not only results in improved energy efficiency but it also extends service life and reduces maintenance cost of equipment.

If inductive reactance ( $X_L$ , in  $\Omega$ ) and reactive power ( $Q$ , in VARs) are encountered through a distribution transformer, application of PF capacitor is recommended at the distribution bus feeding the transformer. When the inductive reactance ( $X_L$ , in  $\Omega$ ) and reactive power ( $Q$ , in VARs) are contributed by large motors, the greatest PF corrective effect and energy loss reduction is realized when the capacitors are installed at each motor load (on line side), or between the motor starters and the motors.

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<sup>1</sup>NFPA® 70E, 2012

However, the initial cost for a number of small banks of capacitors would tend to be higher than the initial cost of a single, large, bank of capacitors, catering to several motor loads. In such cases, the higher initial cost must be weighed against gains in energy cost and PF correction benefits. Systems that experience large load swings, throughout the day or month, are good candidates for automatic switching correction systems. As explained earlier, these systems automatically correct the PF as load variations occur.

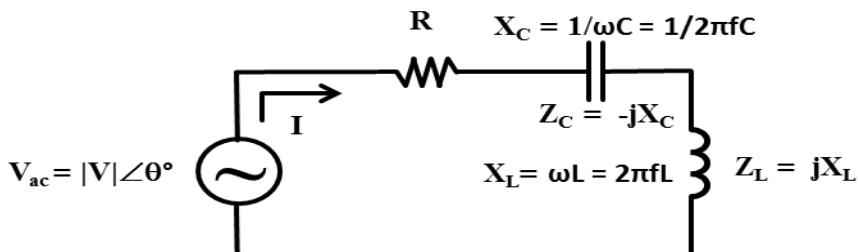
**Addition of inductance or inductive reactance to correct for leading power factor:** If the power factor is less than “1” due to the predominance of capacitance or capacitive reactance,  $X_C$ , inductance or inductive reactance,  $X_L$ , must be added in series or shunt to raise the power factor closer to 100%.

In some AC electrical systems, such as the power transmission lines, the total impedance as seen from the utility side is predominantly capacitive. In such cases, *inductance* must be added to improve the power factor. Addition of power factor correcting inductive reactance, and analytical methods to correct power factor in predominantly capacitive electrical systems, are illustrated in self- assessment problems 2 and 3.

### Frequency manipulation approach for correction of power factor

While pursuit of a perfect *unity* power factor in low power factor situations is not always feasible or practical, for simplicity, we will limit our discussion associated with frequency based power factor correction to scenarios where power factor is, indeed, raised to 1 or 100%. As stated earlier, frequency manipulation method pertains to AC electrical systems where the both inductive and capacitive reactances are present, in addition to resistance.

Consider the AC circuit shown in Figure 3.11 below with an existing power factor that is less than 1.



**Figure 3.11** – An RLC (Resistor, Inductor and Capacitor) circuit.

In order for this circuit's power factor to be unity or 100%, the circuit must be purely resistive. In other words,  $Z_{Eq} = R$  and  $Z_C$  must cancel or offset  $Z_L$ . So, when inductive reactance  $X_L$  and capacitive reactance  $X_C$  are fixed, the electrical frequencies "f" and "ω" may be changed such that the magnitudes of  $Z_C$  and  $Z_L$  become equal and cancel each other. The frequency at which  $|Z_C|$  and  $|Z_L|$  equate is called **resonant frequency**,  $f_0$  or  $\omega_0$ . An electrical circuit operating at resonance frequency is called a **resonant circuit**. The derivation and mathematical proof for resonance frequency is as follows:

In order for  $Z_L$  and  $Z_C$  to cancel each other when added together in a series combination:  $Z_L = -Z_C$

$$\text{Since } Z_L = j2\pi fL \text{ and } Z_C = -jX_C = -j\frac{1}{2\pi fC}$$

$$j2\pi fL = -\left(-j\frac{1}{2\pi fC}\right)$$

$$\text{Or, } X_L = X_C \quad \text{Eq. 3.7}$$

$$2\pi fL = \frac{1}{2\pi fC} \text{ and, } f^2 = \frac{1}{(2\pi)^2 LC}$$

And,

$$f = f_o = \frac{1}{2\pi\sqrt{LC}}, \text{ or, since } 2\pi f = \omega \quad \text{Eq. 3.8}$$

$$\omega = \omega_o = \frac{1}{\sqrt{LC}} \quad \text{Eq. 3.9}$$

Where,

L = Inductance in henry, or H

C = Capacitance in Farads, or F

ω = Angular frequency, in general, in radians/sec.

ω<sub>0</sub> = Resonance angular frequency, in radians/sec.

f = Electrical frequency, in cycles/sec, hertz, or Hz; 60Hz in the US.

f<sub>0</sub> = Resonance electrical frequency, in hertz.

Z<sub>C</sub> = Impedance contribution by capacitance in an AC circuit, in Ω.

Z<sub>L</sub> = Impedance contribution by inductance in an AC circuit, in Ω.

X<sub>C</sub> = Reactance due to capacitance in an AC circuit, in Ω.

X<sub>L</sub> = Reactance due to inductance in an AC circuit, in Ω.

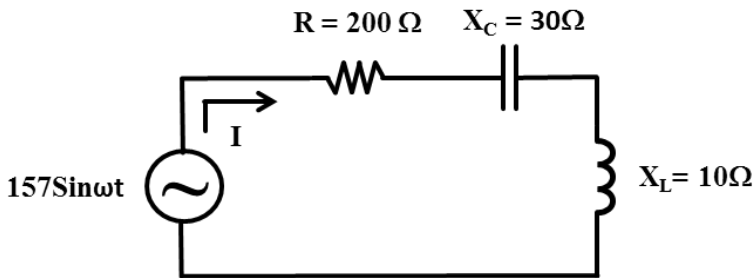


As obvious, the frequency method for correcting power factor is feasible only in circumstances where the electrical frequency can be varied through the use of electromechanical systems such as electrical power generators, or via electronic variable frequency drives.

**Example 3.7**

The output of a variable frequency drive, as shown in the circuit below, is  $157\sin\omega t$ . The VFD output is currently set at 60Hz. This drive is connected to a resistive load, capacitive reactance, and an inductive reactance.

- (a) What should be the new frequency setting to attain a power factor of 1, or 100%?
- (b) What is the resonance frequency for this AC system?
- (c) Could a power factor other than unity be attained through variation of frequency?



**Solution**

(a) As explained earlier in this section, the power factor of an AC circuit, consisting of inductive and capacitive reactance, peaks to the maximum value of unity, or 100%, at resonance frequency,  $f_0$ . However, in this case, because the inductance and capacitance are represented as inductive and capacitive reactances, respectively, we cannot apply Eq. 3.6, directly, to compute  $f_0$ . We must convert reactances  $X_C$  and  $X_B$  to corresponding capacitance,  $C$ , and inductance,  $L$ , values.

Since  $X_L = 2\pi fL$ ,  $L = \frac{X_L}{2\pi f}$ , or,  $L = \frac{10\Omega}{2\pi(60\text{hz})} = 0.02654 \text{ H}$

And,

$$X_C = \frac{1}{2\pi fC}, \quad C = \frac{1}{2\pi fX_C}, \quad \text{or,} \quad C = \frac{1}{2\pi(60\text{hz})(30\Omega)} = 88.46 \times 10^{-6} \text{ F}$$

$$f = f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2(3.14)\sqrt{(0.02654).(88.46 \times 10^{-6})}} \quad \text{Eq. 3.8}$$

$$\therefore f = f_o = 104\text{hz}$$

(b) As explained earlier, resonance frequency *is* the AC frequency at which the total capacitive reactance in an AC circuit cancels or offsets the total inductive reactance. Therefore, resonance frequency,  $f_0$ , for the given AC circuit is the frequency calculated in part (a). In other words,  $f_0 = 104\text{Hz}$ .

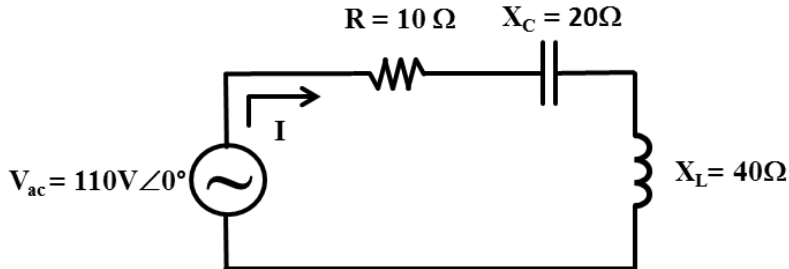
(c) The power factor does, indeed vary with frequency, with all other entities held constant. For instance, with all parameter held constant, if the frequency is lowered to 30Hz, the power factor drops down to, approximately, 96%.

**Ancillary:** Reader is encouraged to prove this. Also see self-assessment problem 4.

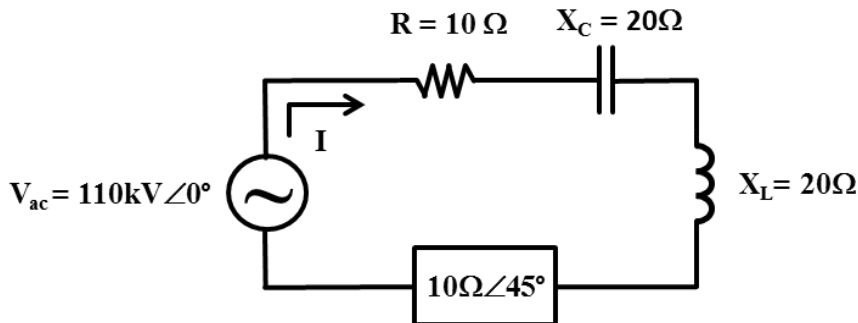
**Segment 3**

**Self-assessment Problems & Questions**

1. Determine the power factor of the circuit shown below, as seen by the AC source.



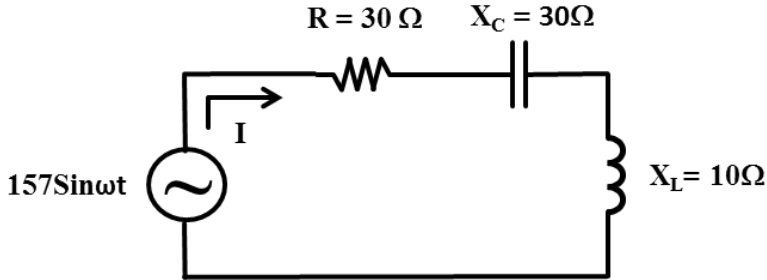
2. Assume that the circuit depicted below represents one phase of a special power transmission line. Determine the power factor of the circuit shown below, as seen by the AC source.



3. If the power factor in problem 2 is less than 1.0, how much capacitance or inductance must be added in series to raise the power factor to unity?

4. The output of a variable frequency drive, as shown in the circuit below, is  $157\sin\omega t$ . The VFD output is currently set at 50Hz. This drive is connected to a resistive load, capacitive reactance, and an inductive reactance.

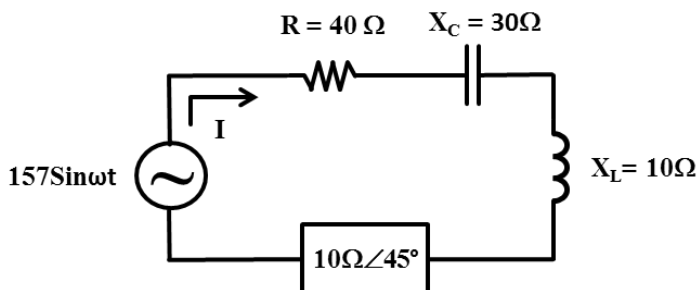
- (a) What should be the new frequency setting to attain a power factor of 1, or 100%.
- (b) What is the existing power factor, at 50Hz?
- (c) What would be the power factor if all circuit elements remain unchanged and the VFD frequency is lowered to 30Hz.



5. The HMI (Human Machine Interface) monitor of an Automated HVAC system, monitoring an air washer supply fan motor is indicating a reactive power,  $Q_1$ , of 60kVARs. This system is located in the United Kingdom, where the AC frequency is 50 Hz. Determine the amount of capacitance that must be added to improve the power factor of the motor branch circuit such that the reactive power is reduced to  $Q_2$  of 20kVARs. The branch circuit is operating at 240  $V_{RMS}$ .

**6. Power Factor Improvement and Cost Savings:** In conjunction with the local utility company DSM program, a manufacturing plant is being offered \$2 per kVA for improvement in power factor from 0.75 to 0.85. The plant is operating at its contract level of 30 MW. Determine estimated annual pre-tax revenue if the plant accepts the offer.

7. The output of a variable frequency drive, as shown in the circuit below, is  $157\text{Sin}\omega t$ . The VFD output is currently set at 60Hz. This drive is connected to a resistive load, capacitive reactance, inductive reactance and a “black box” load,  $Z_B$ , of  $10\Omega\angle 45^\circ$ . What should be the new frequency setting to attain a power factor of 1, or 100%?



# **APPENDICES**

## **Appendix A Solutions for Self-Assessment Problems**

This appendix includes the solutions and answers to end of segment self-assessment problems and questions.

**MADE AVAILABLE UPON PURCHASE OF COURSE**

## **Appendix B Common Units and Unit Conversion Factors**

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## **Appendix C Greek Symbols Commonly Used in Electrical Engineering**

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